

## Take Another 20% Off!

Objective: Apply consecutive growth and/or decay factors to problems involving two or more percent changes.

Your friend found a coupon in today's newspaper for 20% off at Old Navy. The \$100 jacket she had been eyeing all season was already reduced by 40%. She clipped the coupon, drove to the store, selected her jacket and walked up to the register. The cashier brought up a price of \$48; Your friend thought that the price should have been only \$40 and began throwing a fit. The store manager arrived to calm her down, offered to reenter the sales transaction, and again calculated the sales price to be \$48. Your friend turned red in the face and stomped out of the store vowing never to return. She went to your house to tell you about the scam at Old Navy.

1. How do you think your friend calculated a price of \$40?
2. You grab a pencil and start your own calculation. You determine the decay factor and use it to calculate the ticketed price that reflects the 40% reduction. Show these calculations to calculate the price on the jacket.
3. Use the decay factor associated with the 20% off coupon to determine the price the cashier and manager both determined was the final price of the jacket.

Now you are curious if you could justify a better price by applying the discounts in the reverse order. You start a new set of calculations, again using the decay factor approach.

4. Starting with the list price, determine the sale price after taking the 20% deduction.
5. Apply the 40% discount to the intermediate sale price.
6. Which sequence of discounts gives a better sale price?

### Consecutive Decay Factors

The important point to understand from Problems 1 through 6 is that multiple discounts are always applied sequentially...one after the other. That is, discounts are never added together first before calculating the discounted price. With this in mind, you can determine the sale price simply by using the associated decay factors.

The following example shows how applying multiple discounts as decay factors can be done in a single chain of multiplications.

7. The world's most expensive umbrella ever sold was an Italian designed umbrella made from water resistant crocodile skin. The umbrella is guaranteed not to turn inside out with a large gust of wind but the gust may pull you off your feet. If not the gust, perhaps the price: \$50,000. Recently the umbrellas haven't been selling so well so over several weeks there were several discounts:  
*20% off list*                      *30% off new price*                      *an additional 40% off every umbrella*

a. determine the final price of one of these umbrellas.

b. what is the overall percent decrease of the umbrella from its original price?

c. calculate the final price in reverse order. Is the final price the same as in part a.?

### **Forming a Single Decay Factor for Consecutive Percent Decreases**

A series of decay factors multiplied together can be used to obtain a single decay factor. This single decay factor represents all of the decay factors in the series of reductions.

Another way to say this is that the effect of applying 20%, 30% and then 40% consecutive discounts is identical to a single discount of 66.4%. The single discount is called the *effective discount* and the associated single decay factor is called the *effective decay factor*.

8. a. Determine the effective decay factor that represents the cumulative effect of the consecutively applying Old Navy's 40% and 20% discounts.

b. Use this decay factor to determine the effective discount on Maureen's jacket.

9. a. Determine the effective decay factor that represents the cumulative effect of consecutively applying discounts of 40% and 50%.

b. Use the decay factor to determine the effective discount.

### **Consecutive Growth Factors**

You can also use the single chain of multiplications approach to apply multiple percent increases. In this case, you will multiply by growth factors. The following problem will guide you in using this method for such growth.

10. Your business is growing faster than you had ever imagined. Last year you had 100 employees statewide. This year, you opened several additional locations and increased the number of workers by 30%. With demand so high, next year you will be opening new stores nationwide and plan to increase your employee roll by an additional 50%.

To determine the projected number of employees next year, you can use growth factors to simplify the calculations.

a. Determine the growth factor corresponding to a 30% increase.

b. Apply this growth factor to calculate your current workforce.

c. Determine the growth factor corresponding to an additional 50% increase.

d. Apply this grow factor to your current workforce to determine the projected number of employees next year.

e. Write a single chain of multiplications to calculate the projected number of employees from last year's workforce of 100. Compare your answer to the result in part d.

You can form a single growth factor to represent two or more consecutive percent increases. The single growth factor is the product of the associated growth factors. In the last problem, the single growth factor is given by the product  $1.30 \cdot 1.50$ , which equals 1.95. This shows that the number of employees nearly doubled since last year. Note also that the associated percent increase is calculated by subtracting 100% from the single growth factor 195% to obtain 95%. Therefore the effect of applying consecutive 30% and 50% increases is identical to a single increase of 95%. The single percent increase is called the *effective increase*. and the associated single growth factor is called the *effective growth factor*.

11. A college laboratory technician started his job at \$30,000 and expects a 3% increase in his salary each year.
- What is the growth factor associated with the percent increase expected?
  - What is the technician's salary after 3 years? After 10 years?
  - What is the effective percent increase after 3 years? After 10 years?

### **Forming Single Factors for Consecutive Percent Increases and Decreases**

The next problem will now guide you in applying consecutive growths and decays by using growth and decay factors together.

12. You purchased \$1000 in a recommended stock last year and gleefully watched as it rose quickly by 30%. Unfortunately, the economy turned downward and you discover that your stock has recently fallen 30% from last year's high. The question is, have you made or lost money on your investment?
- What is the growth factor corresponding to a 30% increase?
  - What is the stock worth after the 30% increase?
  - Form the decay factor corresponding to a 30% decrease.
  - What was the stock worth after the 30% decrease?
  - What would the single factor be by combining both the decay and the growth factor (multiplying them)?
  - Is it greater or less than 1? Base on your answer, how could you tell whether the effective factor is an effective growth factor or an effective decay factor?

The last problem shows that you can form a single factor that represents the cumulative effect of applying the consecutive percent increase and decrease—the single factor is the *product* of the growth and decay factors.

13. Is the cumulative effect different if you apply a 30% decrease first and then a 30% increase?

### Exercises

1. A \$300 suit is on sale for 30% off. You present a coupon at the cash register for an additional 20% off.
  - a. Form the decay factor corresponding to each percent decrease.
  
  - b. Use these decay factors to determine the price you paid for the suit.
  
2. Your union has just negotiated a 3-year contract containing annual raises of 3%, 4%, and 5% during the term of contract. Your current salary is \$42,000. What will you be earning in 3 years?
  
3. You had anticipated a large demand for a popular toy and increased your inventory of 1600 by 25%. You were able to sell 75% of your inventory. How many toys remain?
  
4. You deposit \$2000 in a 5-year certificate of deposit that pays 4% interest compounded annually. To the nearest dollar, what balance will the account show when your certificate comes due?
  
5. Budget cuts have severely crippled your department over the last few years. Your operating budget of \$600,000 has decreased by 5% each of the last 3 years. What is your current operating budget?
  
6. When you became a manager, your \$60,000 annual salary increased by 25%. You found the new job too stressful and requested a return to your original job. You resumed your former duties at a 20% reduction in salary. How much are you making at your old job due to the transfer and return?

7. A coat with an original price tag of \$400 was marked down by 40%. You have a coupon good for an additional 25% off.
- What is its final cost?
  - What is the equivalent percent discount?
8. A digital camera with an original price tag of \$500 was marked down by 40%. You have a coupon good for an additional 30% off.
- What is the decay factor for the first discount?
  - What is the decay factor for the additional discount?
  - What is the effective decay factor?
  - What is the final cost of the camera?
  - What is the equivalent percent discount?
9. Your friend took a job 3 years ago that started at \$30,000 a year. Last year, she got a 5% raise. She stayed at the job for another year but decided to make a career change and took another job that paid 5% less than her current salary. Was the starting salary at the new job more, or less, or the same as her starting salary at her previous job?
10. You wait for the price to drop on a diamond-studded watch at Macy's. Originally, it cost \$2500. The first discount was 20% and the second discount is 50%.
- What are the decay factors for each of the discounts?
  - What is the effective decay factor for the two discounts?
  - Use the effective decay factor in part b to determine how much you will pay for the watch.