Goals of the Next Group of Handouts:

1. To accurately express and compute with measured quantities
2. To determine geometrical properties of two- and three-dimensional figures
3. To describe one-variable data numerically and graphically
4. To begin to create and use mathematical models

In the workplace:

An architect makes a scale drawing when designing a building. An automotive engineer creates a three-dimensional computer image to test a new idea. A newspaper uses a graph to display economic trends. Scale drawings, computer images, and graphs are a few examples of what we call a **model.**

A model gives people valuable information about a particular real-world situation. The automotive engineer can examine the airflow around a newly designed car before actually building it.

In an attempt to explain why things happen the way they do or to make predictions about something in the future, people sometimes create **mathematical models.** A mathematical model can take the form of an equation, a table of information, a graph, a diagram, a verbal rule, or sometimes a combination of all these.

We will examine specific types of models that are used by people in their work. We will focus much of our attention on learning the mathematics needed to create such models.

The following section contains an activity in which we begin examining and creating models to help us solve the kinds of problems that people face in the real world.

Measurement: Measuring Length

What You Need to Know

- How to perform basic arithmetic operations
- How to round numbers
- How to identify basic polygon shapes

What You Will Learn

- To estimate lengths of objects using various units
- To state the precision of a measuring instrument
- To express the accuracy of a measurement
- To compute perimeters of some two-dimensional figures

Materials

• Meter Sticks, Yard Sticks or Tape Measures

Measurements of various kinds are a necessary and routine part of many occupations. Some measurements must be very precise. Others can be approximate. And there are times when rough estimates are good enough.

The construction industry is particularly dependent on measurements of size. In order to predict the cost of a construction project, estimates of amounts of concrete, steel, and lumber are needed. These estimates are frequently based on total floor space. To build a house, such things as the length of a gutter or the surface area of a roof or kitchen counter are required.

For budgeting purposes, estimates within 5% or 10% of the actual cost may be sufficient. When ordering materials, greater accuracy is necessary. But the survey that establishes lot boundaries and a carpenter's framing measurements must be made with a high degree of precision.

In job-related tasks that involve quantitative data, clearly expressing accuracy is crucial. In the following activity and throughout the chapter, we will explore ways of determining and communicating the accuracy of measurements.

Activity 1.1 Finding the Dimensions of a Room

Suppose that your classroom is going to be carpeted and repainted. It's also going to get a new heating and air conditioning system. In order to make important and informed decisions about these improvements, measurements must be made.

1. Use a metric ruler or meter stick to measure as accurately as possible the width of a piece of notebook paper.
2. Since *all* measurements are approximate, the *exact* width of your paper cannot be measured. But the more precise your measuring instrument, the closer your measurements are to being exact.

To determine the **precision** of a ruler or meter stick, look at the smallest markings on it. As an example, if you were using a foot-long ruler marked off in sixteenths of an inch, the precision of the ruler would be $\frac{1}{16}$ of an inch.

3. You can indicate the accuracy of a measurement by showing the **maximum likely error** when you write the measurement. To do so, write your result followed by a ± symbol followed by half the precision amount. For example, for a ruler with a precision of $\frac{1}{16}$ of an inch, a measurement might be written as $7\frac{5}{16}$ inches $\pm\frac{1}{32}$ inch. This indicates that the actual length is somewhere between $\left(7\frac{1}{16}\text{ inches} + \frac{1}{32}\text{ inch}\right)$ and $\left(7\frac{1}{16}\text{ inches} - \frac{1}{32}\text{ inch}\right)$.

   Did you measure the width of the paper using the full precision of the meter stick? If not, do so now. Write the width of your notebook paper with an appropriate indication of accuracy.

Units larger or smaller than a meter (or any other metric unit) are indicated with **prefixes** attached to the front of the word. Some of the most common prefixes used in length measurement are listed in **Table 1.1.**

Prefix	Meaning	Unit of Length
kilo-	thousands	1 kilometer = 1 thousand meters
centi-	hundredths	1 centimeter = 1 hundredth of a meter
milli-	thousandths	1 millimeter = 1 thousandth of a meter
micro-	millionths	1 micrometer = 1 millionth of a meter
nano-	billionths	1 nanometer = 1 billionth of a meter

Table 1.1

Sometimes (although rarely) these prefixes are used with non-metric units. Structural engineers often calculate force loads on beams and columns in terms of kilopounds, or thousands of pounds, which are abbreviated kips. And the thickness of a sheet of paper or plastic may be given in mils, which are thousandths of an inch.

4.Now look across your classroom from left to right. Have each person in your group estimate the width of the room in meters. (You may want to examine a meter stick before making your estimate.) How close do you think your estimate might be to the real width of the room?

5. Now measure the room's width using the full precision of the meter stick. Write your result along with an indication of the maximum likely error in the measuring process.

6. Measure the width of the room three more times. Are the four measurements all the same? Explain.

Now examine your group's results. Your four measurements form a **sample data set.** Calculate the **sample mean** (average) width by dividing the sum of all the sample data by the number of measurements in the sample.

8. In a similar manner, determine the front-to-back dimension of the classroom. Find the sample mean for this dimension.

9. Create a geometric model of the floor of your room by making a sketch of the floor and labeling the dimensions with the sample mean length and width.

10. One possible direct use of the linear measurements you have made would be in determining the length of floor molding necessary to cover the edges where the floor meets the walls. Using your sample mean dimensions for the room, calculate the total length of floor molding needed.

    The name of the geometrical property you have just determined is the **perimeter** of the floor, which means the distance around any plane surface (although you may have subtracted doorway widths from your perimeter to find floor-molding length). Since perimeter is a measure of length, it is measured in linear units such as inches, feet, yards, miles, centimeters, and meters.

11. In your own words, write a rule for determining the perimeter of any two-dimensional figure.

12. When finding the perimeter of a figure that you know is a rectangle, explain why it is unnecessary to measure all four sides.

13. In your own words, write a rule for determining the perimeter of a rectangle.

14. When finding the perimeter of a figure that you know is a square, will the rule for finding the perimeter of a rectangle work? Explain. Write a simpler rule for finding the perimeter of a square.

Extend the Activity

15. For each length, (a) state the precision with which the measurement was made and (b) write the length with an appropriate indication of accuracy.

    i. 14.25 m

    ii. $3\frac{7}{16}$ in

    iii. 86 cm

    iv. 42.9 mm

    v. 181.2 mi

16. Three students used the same meter stick to measure the length of a wire. They reported the following measurements: 0.6 m, 0.60 m, 0.600 m. Using what you know about precision, explain the meaning of each measurement.

17. A common way of describing measurement error is to express the error as a percentage of the measured value. If a carpenter's tape measure marked off in $\frac{1}{32}$ of an inch is used to measure a 2-inch length, then the built-in error due to the precision of the tape measure is $\frac{1}{64}$ of an inch.

The ratio of this error to the measured length is $\left(\frac{\frac{1}{64}in}{2in}\right) = \frac{1}{128}$. We could say that the error is 1 part in 128.

The **percent error** in the measurement is found by determining another fraction that is equal to $\frac{1}{128}$ but that has a denominator of 100. (Percent means, literally, per hundred.) To do so, we write a **proportion,** or a mathematical statement that two ratios or fractions are equal:

$$\frac{1}{128} = \frac{\text{Percent Error}}{100}.$$

If both sides of this proportion are multiplied by 100, then the right-hand side will just be the percent error. The lefthand side becomes $(100)\left(\frac{1}{128}\right) \approx .78$. Therefore, the percent error is 0.78%.

As a general rule, we can simplify this process and directly calculate the percent error as follows:

$$\text{Percent Error} = \left(\frac{\text{actual error}}{\text{length measured}}\right)(100)$$

a) If the same tape measure is used to measure a 70-inch length, determine the percent error in the measurement.

b) If a surveyor makes an error of 0.32 meters in measuring an 84-meter distance, what is the percent error in the measurement?

c) The Universal Transverse Mercator (UTM) is one type of global positioning system. Originally developed by the military, it is separated into 60 zones around the earth, 11 of which cover the United States. At the edges of each zone, the accuracy of this GPS system is 1 part in 2500. What percent error does this correspond to?

d) A new type of measuring device uses ultrasonic waves to measure distances. Devices of this type are accurate to 2 inches when used to measure a distance of 50 feet (600 inches). Determine the percent error in this measurement.

e) Determine the inherent percent error in the measurements of the width of your classroom (item 5 of the activity) due to the precision of the measuring device you used.

18. a) Try to estimate (from memory) the dimensions of the room where you sleep. Write down your estimates both in meters and in feet. Then measure the actual dimensions and compare them to your estimates. Did you find that your estimates were generally too high or too low? To improve your ability to make geometrical estimates, try this with other rooms and see if your accuracy increases.

b) Try estimating things like the number of teaspoons in a cup of water. Check to see how close your estimate comes to the correct number.

c) Estimate how many cars, sitting end to end, would reach the length of a mile. Then measure the length of a typical car and check your estimate.

19. Finding the length of floor molding needed for the classroom was mentioned as a reason for determining lengths. Surveyors must make very accurate measurements of property dimensions, since their results are the basis for legal descriptions of lot boundaries. Officials of cities and towns that have ocean borders need to know the lengths of shoreline for which they are responsible. List other jobs that require detailed knowledge of length measurements (including perimeters). Also, identify the kind of accuracy that may be required in each case.