

### Practice Exam 2

This is a set of problems that you should study for the first exam. The problems cover material from **Lecture 6 through Lecture 14**. Problems on the exam will be similar but not guaranteed to be exactly like these problems. Any application problems will be directly “plagiarized” from your math book homework. No Graphing calculators used on the exam. If you need a scientific calculator you can check one out at the MLC and I will return it for you. **Remember to check solutions when solving equations if necessary.**

I've added a page of grids at the end of the practice exam for you convenience.

1. For the one-to-one function  $f(x) = \frac{2x+3}{3x+1}$

a) find the inverse function  $f^{-1}(x)$

b) find the domain and range of  $f(x)$

30. Graph the following by first indicating the basic function and the transformations in the order that they occur. Then show the basic function and the graph of this function.

$$f(x) = 1 - \sqrt{x+3}$$

2. Graph the following piecewise defined function. State the domain and the range.

$$f(x) = \begin{cases} x^2 - 1 & x \leq 0 \\ x + 5 & 0 < x \leq 2 \end{cases}$$

3. Sketch a graph of the following piecewise defined function.

$$f(x) = \begin{cases} x + 2 & x \leq -1 \\ x^3 & -1 < x < 1 \\ -x + 3 & x \geq 1 \end{cases}$$

4. Given the quadratic function  $f(x) = -2x^2 + 6x + 1$

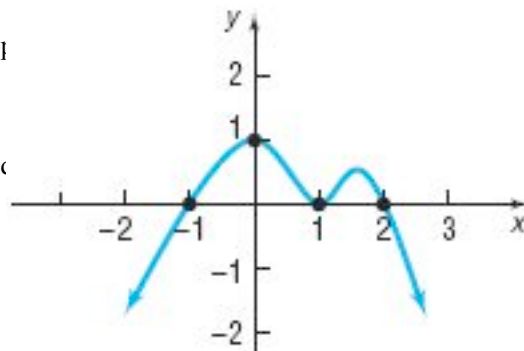
a. write it in the vertex form  $f(x) = a(x-h)^2 + k$

b. state the vertex and whether it is a maximum or minimum point.

c. write down in words, the transformations of the basic function (in the order they happen)

d. graph the  $f(x) = -2x^2 + 6x + 1$  using the transformations listed in part c.

5. Determine a polynomial function  $f(x)$  whose graph is shown below. Leave it in factored form that is necessary to make sure your function has the y-intercept shown in the graph.



6. For the function  $f(x) = \frac{1}{4}(x+1)^2(x-2)^2(x-3)$  find the following:

a. x and y intercepts

b. determine whether graph touches or crosses the x-axis at each x-intercept.

c. End behavior (or global characteristics) of function.

d. maximum number of turning points on the graph.

e. make a sign chart as discussed in class to determine where function is above/below the x-axis

f. graph the function using all information found in the previous parts.

**7. Analyzing the Motion of a projectile:**

A projectile is fired from a cliff 200 feet above the water at an inclination of  $45^\circ$  from the horizontal, with a muzzle velocity of 50 feet per second. The height  $h$  of the projectile above the water is given by

$$h(x) = \frac{-32x^2}{(50)^2} + x + 200$$

where  $x$  is the horizontal distance of the projectile from the location it was fired.

- At what horizontal distance from the place it was fired does the projectile reach its maximum height?
- Find the maximum height.
- At what horizontal distance from the place it was fired does the projectile strike the water?

8. A fourth degree polynomial function  $f(x)$  whose coefficients are real numbers has zeros:  $1, 2, 2+i$  and crosses the  $y$ -axis at  $-2$ . Find the function  $f(x)$ .

9. Find the real and complex zeros of the function  $f(x) = x^4 - 7x^3 + 26x^2 - 10x - 44$  and write  $f(x)$  in factored form.

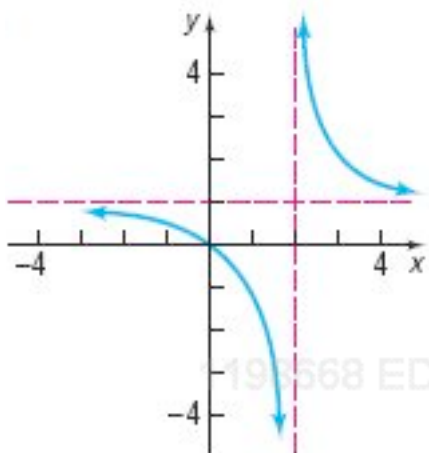
10. Solve the equation  $x^4 - 2x^3 + 10x^2 - 18x + 9 = 0$  (solutions can also be complex numbers)

11. Find  $k$  so that  $f(x) = x^3 - kx^2 + kx + 2$  has the factor  $x - 3$ .

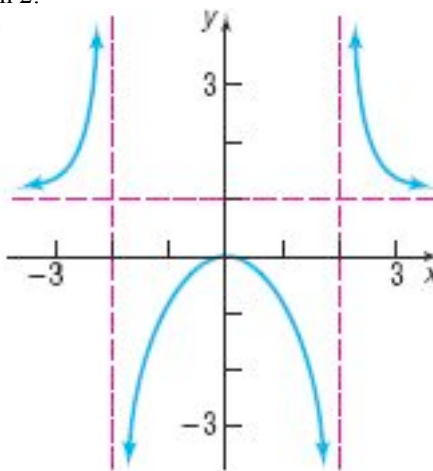
12. Use the graphs provided to find:

- The domain and range
- The intercepts (if any)
- Horizontal asymptotes (if any)
- Vertical asymptotes (if any)
- Oblique asymptotes (if any)

Graph 1:



Graph 2:



13. Graph (the mixed form)  $R(x) = 2 - \frac{1}{x+1}$  using transformations on the graph of  $y = \frac{1}{x}$  as discussed in class. Be sure to state what the transformations are and the order in which they will be applied.

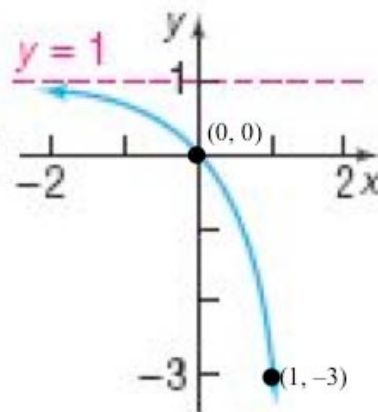
14. For the function  $f(x) = \frac{x^2 - x - 12}{x^2 - 9}$  find the following:

- Domain
- Range
- Coordinates of any holes in graph
- Vertical Asymptotes (if any)
- Horizontal Asymptotes (if any)
- $x$  and/or  $y$  intercepts (if any)
- sign chart for function to determine where function is above/below  $x$ -axis

and then graph  $f(x)$  using the information found in the previous parts. Label intercepts, holes, and asymptotes on the graph.

15. Find an exponential function that has the following graph:

remember the graph of the basic exponential function and determine what transformations need to be done to get the graph shown



16. For the function  $f(x) = 2\left(\frac{1}{3}\right)^x - 1$

- graph the function using transformations of the basic function (hint: 1st convert the fraction to a power of 3)
- graph it's inverse function using symmetry over  $y=x$  line
- determine the inverse function  $f^{-1}(x)$  algebraically

17. Application of Exponential functions:

**Radioactive Decay** Strontium 90 is a radioactive material that decays according to the function  $A(t) = A_0e^{-0.0244t}$ , where  $A_0$  is the initial amount present and  $A$  is the amount present at time  $t$  (in years). Assume that a scientist has a sample of 500 grams of strontium 90.

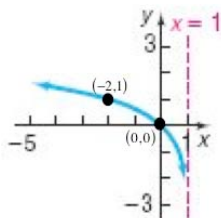
- What is the decay rate of strontium 90?
- How much strontium 90 is left after 10 years?
- When will 400 grams of strontium 90 be left?
- What is the half-life of strontium 90?

18. Find the domain of the function  $\log_3\left(\frac{x}{x-1}\right)$

19. Find the exact value of  $\log_{\frac{1}{3}} 9$  without using a calculator by using the change of base formula for logarithms.

20. Find a logarithmic function that has the following graph:

remember the graph of the basic logarithmic function and determine what transformations need to be done to get the graph shown



21. Use the function  $f(x) = 2 - \log(x-3)$  to determine the following:

- Find the domain of  $f$
- Graph  $f$
- From the graph, determine the range and any asymptotes of  $f$ .
- Find  $f^{-1}$ , the inverse of  $f$ .
- Use  $f^{-1}$  to find the range of  $f$ .
- Graph  $f^{-1}$

19. Solve each of the following equations

a.  $\log_5(x^2 + x + 4) = 2$

b.  $8 \cdot 10^{2x-7} = 3$

20. Expand the expression into a sum and/or difference of logarithms:  $\log \left[ \frac{x(x+2)}{(x+3)^2} \right], x > 0$

21. Solve the following logarithmic equation (write answer in terms of  $e$  if it is the log's base)

a.  $\ln x - \ln(x+2) = 4$

b.  $\log_2 x^{\log_2 x} = 4$

c.  $\log_2(x+1) - \log_4 x = 1$  (hint: use change of base formula for logs first)

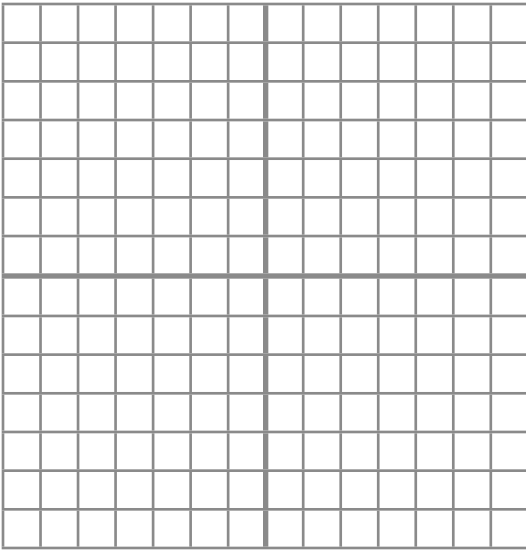
22. Solve the following exponential equation. Don't approximate answers.

a.  $\left(\frac{4}{3}\right)^{1-x} = 5^x$

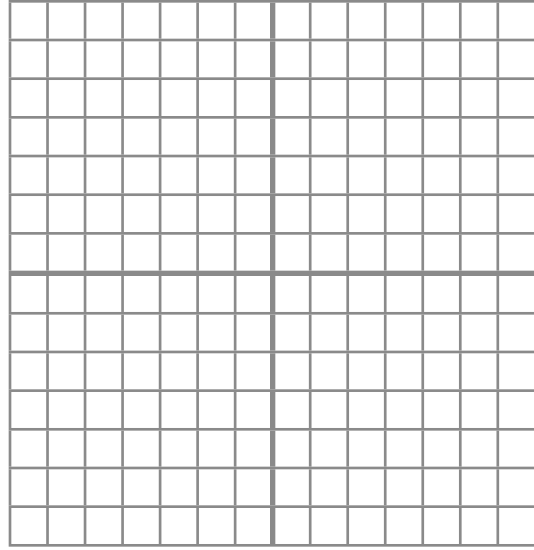
b.  $3^x - 14 \cdot 3^{-x} = 5$  (hint: multiply both sides by  $3^x$  first)

c.  $\frac{e^x - e^{-x}}{2} = -2$

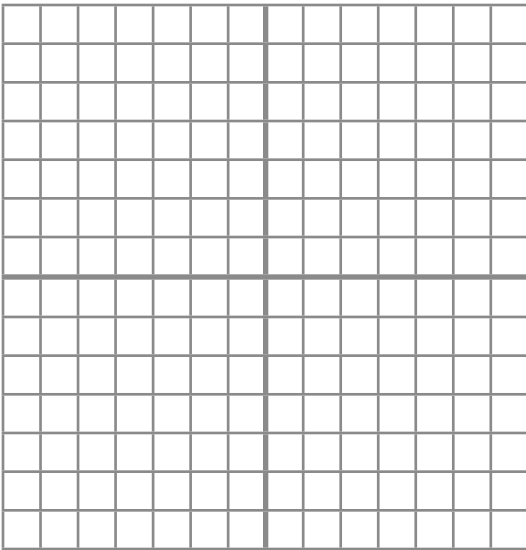
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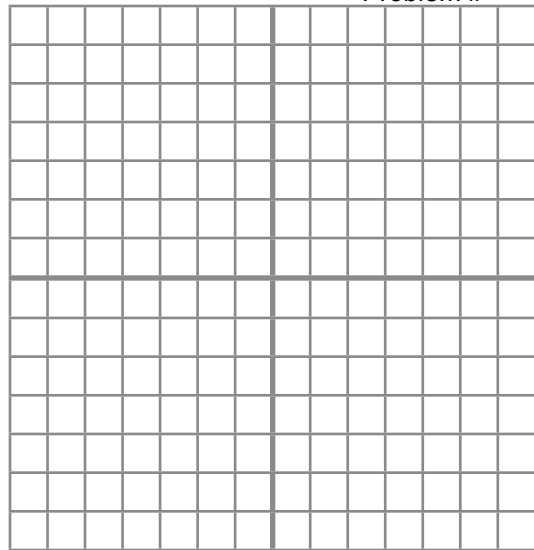
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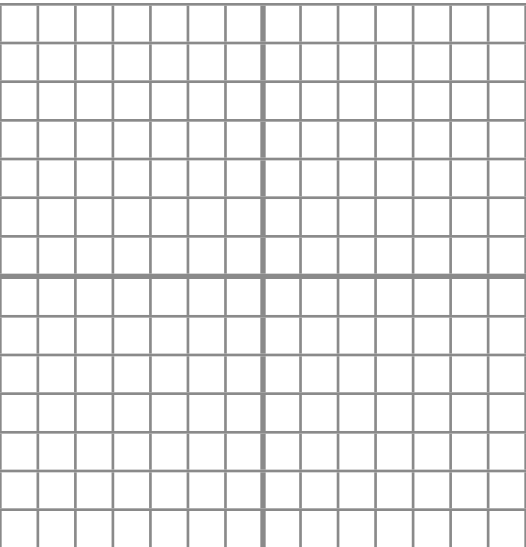
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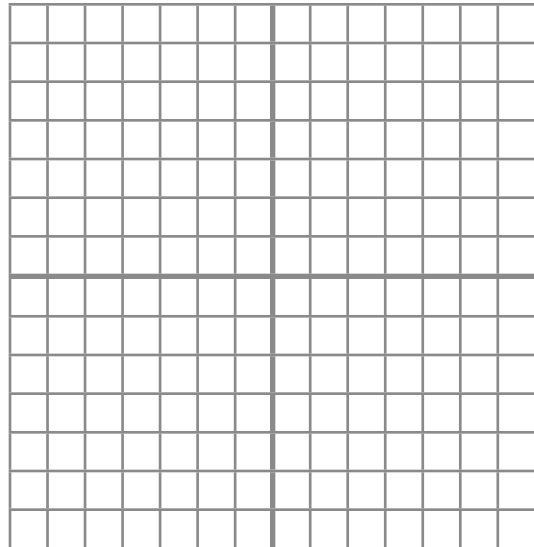
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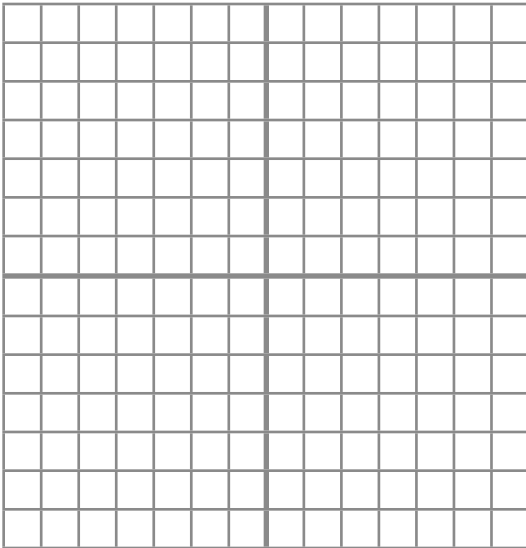
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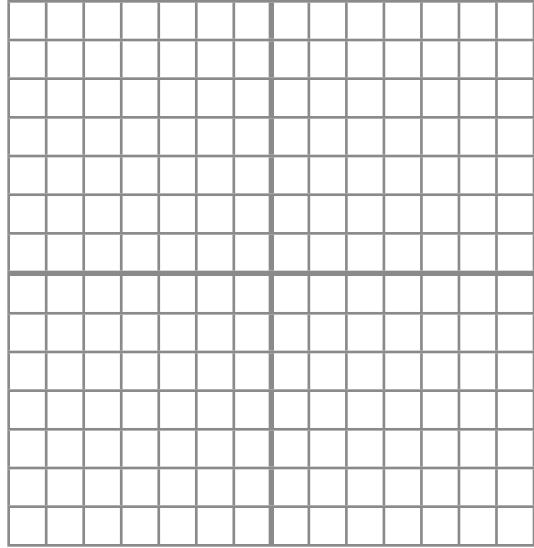
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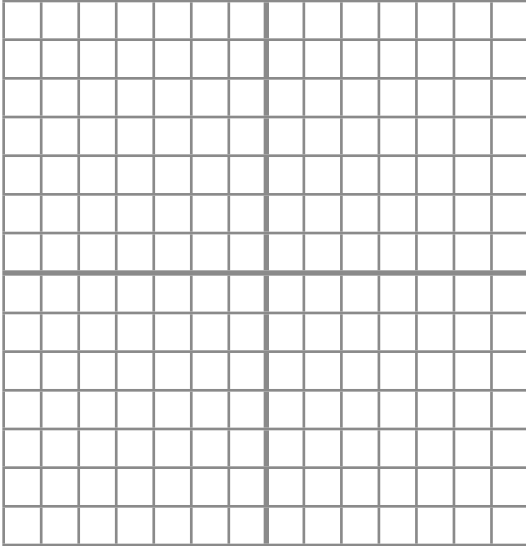
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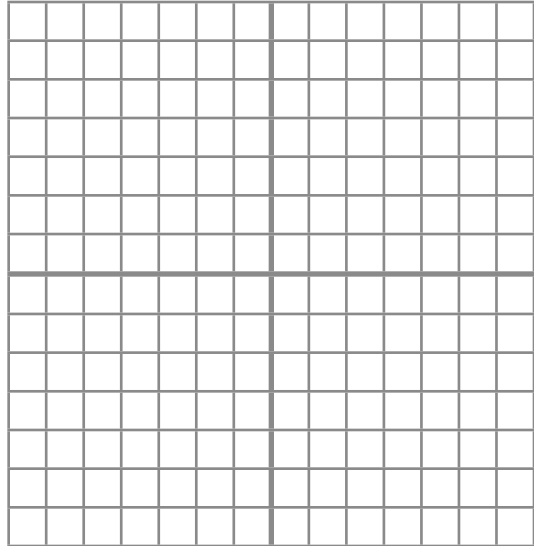
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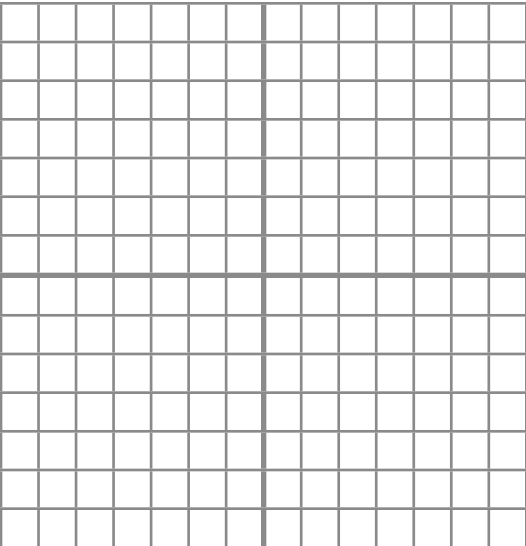
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