Math 4 Pre-Calculus
Lecture 1, Part 4

Name_________________________
Date_________________________

Factoring — R.5 and R.8

Homework:
(do after section covered in lecture)
pg. 56: 5 to 101 eoo
pg. 78, 79: 75 to 99 eoo

When asked to factor an expression, there are many different approaches. Below are a few questions to ask yourself before you begin to factor the expression.

1. Do all of the terms in the expression have any common factors?
   a) If they do, factor out the **Greatest Common Factor (GCF)**.
   b) If they don't, move on to the next question.

2. Does the expression have **four terms**?
   a) If there are four terms, try to factor by grouping.
   b) If factoring by grouping doesn't work, try factoring three of the four terms into a perfect square, in order to get a difference of squares, difference of cubes or sum of cubes. Move on to question 4.
   c) If there are not four terms, move on to the next question.

3. Does the expression have **three terms**?
   a) If there are three terms, determine the leading coefficient.
      – If the leading coefficient is 1 and the expression is in standard form (with each term decreasing in power), try *factoring it into two binomials* where the first terms in the binomial are factors of the first term in the trinomial and the second terms in the binomials are factors of the last term of the trinomial that add to the coefficient of the middle term.
      – If the leading coefficient is **not** 1 and the expression is in standard form (with each term decreasing in power), try *factoring it into two binomials* where the first terms in the binomial are factors of the first term in the trinomial and the second terms in the binomials are factors of the last term of the trinomial, and the middle term of the trinomial is the sum of the products of the outer and inner terms of the binomials. You might have to use "trial and error" if your first attempt isn't correct.
      – If the leading coefficient is **not** 1 and the expression is in standard form (with each term decreasing in power), AND you feel there are too many choices for the "trial and error" method, use the "a-c grouping method". Separate the middle term of the trinomial into two terms whose coefficients are factors of the product of ac and whose sum is the coefficient of the original middle term. You should now have an expression with four terms, so try grouping.
   b) If there are not three terms, move on to the next question.

4. Does the expression have **two terms**?
   a) If there are two terms, check to see if the expression is a *difference of squares*. For example, $a^2 - b^2$.

– If it is a *difference of squares*, factor the expression into two binomials that have opposite operations. **Note**: a sum of squares is irreducible over the set of real numbers —unless it has a GCF. The formula is

$$a^2 - b^2 = (a-b)(a+b)$$

b) If there are two terms, check to see if the expression is a *difference or a sum of cubes*. For example, $a^3 - b^3$ or $a^3 + b^3$.

– If it is a *difference or a sum of cubes*, factor the expression into a binomial and a trinomial using the appropriate formulas below. The formulas are

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

c) If there are not two terms, move on to the next question.

5. Is the expression **factored *completely***? In other words, can any of the factors be factored further?
   a) If any one of the factors can be factored further, repeat questions 1–4 with that factor.
   b) If the expression is factored completely, move on to the next question.

6. Have you checked your answer by multiplying it out to see if the product is the same as the original expression?

Hints:

◊ Remember that some expressions are irreducible over certain sets of numbers. For example, $x^2 - 15$ is irreducible over the set of rational numbers, but can be factored into $\left(x - \sqrt{15}\right)\left(x + \sqrt{15}\right)$ over the set of real (rational and irrational) numbers.

◊ Pay attention to the directions! For section 1.3, "Factor" means factor over the set of rational, real numbers.

◊ A sum of squares are irreducible over the set of real numbers—unless the two terms have a GCF. For example, the expression, $x^2 + 16$, is irreducible over the set of real numbers, but can be factored into $(x - 4i)(x + 4i)$ over the complex numbers.

◊ Look for patterns of perfect square trinomials. The formulas are

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

◊ If any terms have negative exponents then still check for common factors between terms. If there are any, factor out the **lowest power** of the common factor. In some cases this power may be a negative number!

Factor.

1. $2x^2y - y$
2. $2xy - y + 4x - 2$
3. $x^2 - 5x - 6$
4. $2x^2 + 7x + 3$
5. $x^2 - 9$ $x^2 + 9$ $x^2 - 10$
6. $x^3 - 8$
7. $x^3 + 64$

More Factoring:

Factor the polynomial.

8. $16x^5y^2 + 8x^3y^3$
9. $2(x+3)^2 + x(x+3)$
10. $x^7 + 27xy^6$
11. $5x^3 + 10x^2 - 20x - 40$
12. $y^2 + 9 - 6y - 4x^2$
13. $81x^8 - 16$
14. $9x^2 + 24x + 16$
15. $3x^2 - 4x + 2$

16. $8c^6 + 19c^3 - 27$

17. $(x-5)^{-1}(6) + 4(x-5)^{-3}$

18. $(4x-1)^{-1/2}\left(\frac{1}{2}\right) + 4(4x-1)^{1/2}$