Math 4 Pre-Calculus
Lecture 2, Part 1

Name_________________________
Date__________________________

Homework:
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Rational Expressions — R.7

Fractional Expressions

A fractional expression is a quotient of two algebraic expressions.

Rational Expressions

A rational expression is a specific kind of fractional expression, where the quotient is made up of two polynomials.

Reminders:

- Before canceling factors when reducing or multiplying fractional expressions, make sure to factor first. An LCD is not necessary when multiplying. Just *cancel factors common to a numerator and a denominator* when multiplying.

- When dividing two fractional expressions, the very first thing to do is change the problem to multiplying the dividend (first expression) by the reciprocal of the divisor (second expression). Do not cancel, until after you flip and factor. An LCD is not necessary when dividing.

- Find and keep the LCD when adding and subtracting fractional expressions. Do not cancel any factors when adding or subtracting, unless you are reducing the final answer. Multiply each term by "missing"/ "missing". Remember to distribute the minus sign to the numerator following a subtraction sign.

- To simplify a complex fractional expression, do one of two methods:
  1. Combine the top, combine the bottom, then divide.
  2. Multiply the entire expression by $\frac{\text{LCD}}{\text{LCD}}$; this means, multiply each term on the top by $\frac{\text{LCD}}{1}$ and each term on the bottom by $\frac{\text{LCD}}{1}$.

Simplify the expression.

1. $\dfrac{2x^2+9x-5}{3x^2+17x+10}$

2. $\dfrac{16-x^2}{x^3-64}$

3. $\dfrac{4x^2-9}{2x^2+7x+6}\cdot\dfrac{4x^4+6x^3+9x^2}{8x^7-27x^4}$

4. $\dfrac{a^3 - 8}{a^2 - 4} \div \dfrac{a}{a^3 + 8}$

5. $\dfrac{5}{x} - \dfrac{2x - 1}{x^2} + \dfrac{x + 5}{x^3}$

6. $\dfrac{2x + 6}{x^2 + 6x + 9} + \dfrac{5x}{x^2 - 9} + \dfrac{7}{x - 3}$

7. $\dfrac{\frac{r}{s} + \frac{s}{r}}{\frac{r^2}{s^2} - \frac{s^2}{r^2}}$

8. $\dfrac{\frac{x + 2}{x} - \frac{a + 2}{a}}{x - a}$

9. $$\frac{\frac{3}{(x+h)^2} - \frac{3}{x^2}}{h}$$

Rationalizing the Denominator

Rationalizing the denominator of an expression means to make the denominator into a rational number. For a fractional expression with two terms in the denominator, this often means multiplying the entire expression by the $\frac{\text{"conjugate of the denominator"}}{\text{"conjugate of the denominator"}}$.

A conjugate of an expression with two terms of the form $a + \sqrt{b}$ is $a - \sqrt{b}$.
A conjugate of an expression with two terms of the form $\sqrt{a} + b$ is $\sqrt{a} - b$.
A conjugate of an expression with two terms of the form $\sqrt{a} + \sqrt{b}$ is $\sqrt{a} - \sqrt{b}$.

Rationalize the denominator.

10. $$\frac{16x^2 - y^2}{2\sqrt{x} - \sqrt{y}}$$

Rationalizing the Numerator

Rationalizing the numerator of an expression means to make the numerator into a rational number. For a fractional expression with two terms in the numerator, this often means multiplying the entire expression by the $\frac{\text{"conjugate of the numerator"}}{\text{"conjugate of the numerator"}}$.

Rationalize the numerator.

11. $$\frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$$

Express as a sum of terms of the form ax^r where r is a rational number.

12. $$\frac{\left(x^3+2\right)^2}{\sqrt{x}}$$

Express as a quotient.

13. $x^{-2/3}+x^3$

Simplify. Write your final answer without negative exponents—rational exponents are ok.

14. $(6x-5)^{-3}(2)\left(x^2+4\right)(2x) \;+\; \left(x^2+4\right)^2(-3)(6x-5)^{-4}(6)$

15. $\left(x^2+9\right)^4\left(-\dfrac{1}{3}\right)(x+6)^{-4/3} \;+\; (x+6)^{-1/3}(4)\left(x^2+9\right)^3(2x)$

16. $$\frac{\left(x^2-1\right)^4(2x) \;-\; x^2(4)\left(x^2-1\right)^3(2x)}{\left(x^2-1\right)^8}$$

17. $$\frac{\left(1-x^2\right)^{1/2}(2x) \;-\; x^2\left(\tfrac{1}{2}\right)\left(1-x^2\right)^{-1/2}(-2x)}{\left[\left(1-x^2\right)^{1/2}\right]^2}$$