

Lines — 2.3

Slope

All linear equations in two variables have a constant rate of change or slope. The slope of a line is a ratio of the vertical change to the horizontal change between two selected points on a line. In other words, slope is $\frac{\text{vertical change}}{\text{horizontal change}}$ or

$$\frac{\text{rise}}{\text{run}} \quad \text{or} \quad \frac{\text{the change in the y-values}}{\text{the change in the x-values}} \quad \text{or} \quad \frac{\text{the change up or down}}{\text{the change left or right}}$$

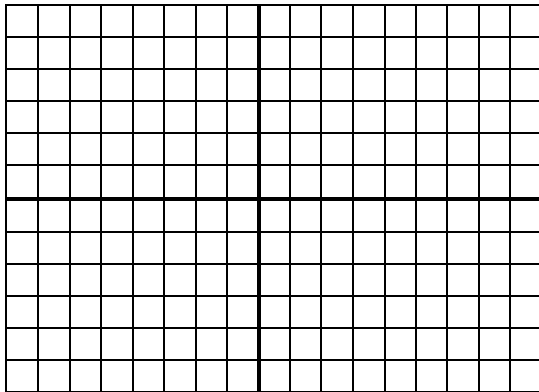
The slope of a line can also be found by using the slope formula. If two points on the line are labeled the first ordered pair: (x_1, y_1) and the second ordered pair: (x_2, y_2) , then the slope of the line can be found by the following formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

It does not matter which ordered pair is labeled the first ordered pair and which is labeled the second ordered pair.

Sketch the line through A and B, and find its slope, m.

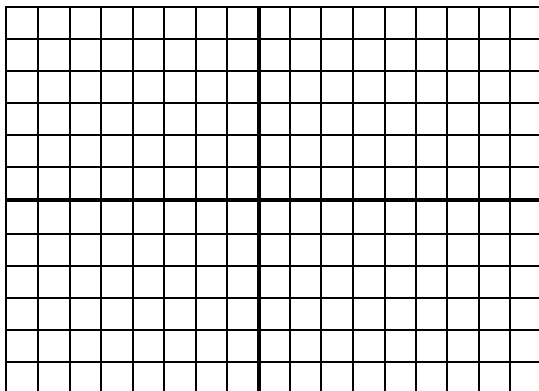
1. $A(4, -1) B(-6, -3)$



*By counting **the units of rise**—vertical (y) change—and **the units of run**—horizontal (x) change—between any two points:*

By using two points on the line and the slope formula:

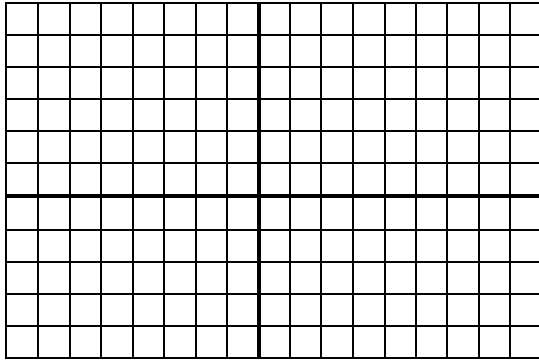
2. $A(4, 3) B(-2, 3)$



*By counting **the units of rise**—vertical (y) change—and **the units of run**—horizontal (x) change—between any two points:*

By using two points on the line and the slope formula:

3. $A(-3, 5)$ $B(-3, -1)$



By counting **the units of rise**—vertical (y) change—and **the units of run**—horizontal (x) change—between any two points:

By using two points on the line and the slope formula:

- A **horizontal line** has a slope of 0. The equation of a horizontal line is $y =$ "the y -value of the y -intercept". For example $y = -2$ or $y = \frac{3}{5}$.
- A **vertical line** has an *undefined* slope. The equation of a vertical line is $x =$ "the x -value of the x -intercept". For example $x = -2$ or $x = \frac{3}{5}$.
- The slopes of **parallel lines** are the same.
- The slopes of **perpendicular lines** are opposite reciprocals.

Slope-Intercept Form $y = mx + b$

If an equation of a line is solved for y , and therefore in the form $y = mx + b$, the **slope of the line** is the coefficient of x or **m** , and the **y -value of the y -intercept is b** . Not all equations of lines are written in this form; however you can solve any equation of a non-vertical line for y and get it in this form.

$y = mx + b$ is called the **slope-intercept form** of an equation of a line because the slope and the y -value of the y -intercept is readily seen.

Point Slope Form $y - y_1 = m(x - x_1)$

If an equation of a line is in the form $y - y_1 = m(x - x_1)$, the **slope of the line** is **m** , and (x_1, y_1) is a point on the line. Any point on the line may be used for (x_1, y_1) . Usually equations that are given are not written in this form; however, this form is very useful for writing an equation of a line.

General Form $ax + by = c$

Any equation in the form $ax + by = c$, is a line, and conversely, every line is the graph of a linear equation.

6. $A(12, -5)$; through $B(0, 4)$

7. $A(7, 3)$; perpendicular to the line $4x + 2y = 9$

8. In 1870 the average ground temperature in Paris was 11.8°C . Since then it has risen at a nearly constant rate, reaching 13.5°C in 1969.

a) Express the temperature T (in $^{\circ}\text{C}$) in terms of time x (in years), where $x = 0$ corresponds to the year 1870 and $0 \leq x \leq 99$

b) During what year was the average ground temperature 12.5°C ?