Homework:
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Math 4 Pre-Calculus
Lecture 4, Part 3

Intercepts, Symmetry and Circles — 2.2 and 2.4

Graphing an equation in x and y means showing a "picture" of all of the points $(x\ ,\ y)$ that satisfy the equation. One way to graph an equation is to plot points. Throughout the rest of this course, we will be looking at various other ways to graph an equation, but for now, just plot points.

Definitions

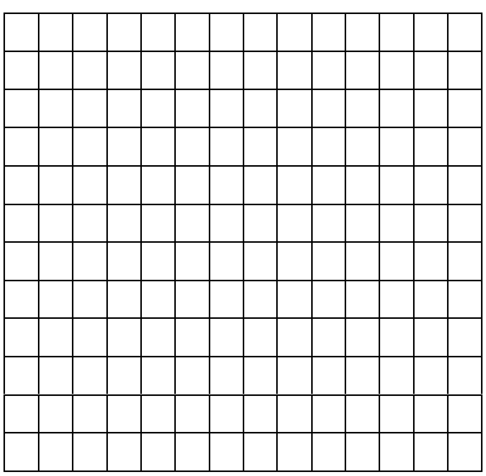
An **x-intercept** of an equation is a point where the graph crosses the x-axis. The x-intercept's y-value is always 0. In other words, the x-intercept is always a point of the form **(? , 0)**. To find the x-intercept of an equation, plug in 0 for y and then find the x-value that goes with y = 0. *Remember, to find an x-intercept, solve for an x-value.*

A **y-intercept** of an equation is a point where the graph crosses the y-axis. The y-intercept's x-value is always 0. In other words, the y-intercept is always a point of the form **(0 , ?)**. To find the y-intercept of an equation, plug in 0 for x and then find the y-value that goes with x = 0. *Remember, to find a y-intercept, solve for a y-value.*

Symmetry
A graph is symmetric with respect to the y-axis if for all points $(x\ ,\ y)$ on the graph there are also the points $(-x\ ,\ y)$. For example, if the point $(3\ ,\ -2)$ is on the graph, then the point $(-3\ ,\ -2)$ is also on the graph. Algebraically, the substitution of *–x* for *x* leads to the same equation. A function that is symmetric about the y-axis is called an **even** function.

Example:

Graphically



Algebraically:

$y = x^2 - 3$

To show the graph of the above equation would be symmetric about the y-axis, plug in –x for x and simplify.

$y = (-x)^2 - 3$

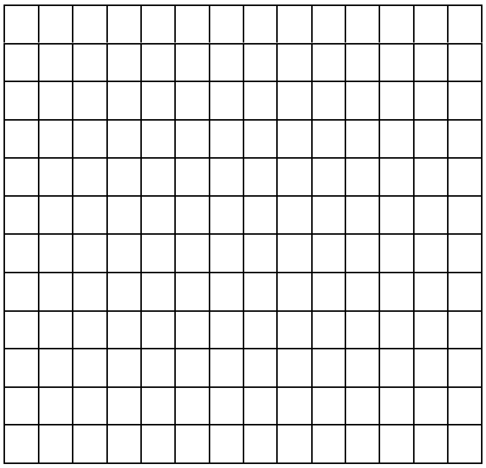
$y = x^2 - 3$

Since the equations are the same, the graph will be symmetric about the y-axis.

A graph is symmetric with respect to the x-axis if for all points $(x\ ,\ y)$ on the graph there are also the points $(x\ ,\ -y)$. For example, if the point $(3\ ,\ -2)$ is on the graph, then the point $(3\ ,\ 2)$ is also on the graph. Algebraically, the substitution of $-y$ for y leads to the same equation.

Example:

Graphically



Algebraically:

$y^2 = x - 1$

To show the graph of the above equation would be symmetric about the x-axis, plug in –y for y and simplify.

$(-y)^2 = x - 1$

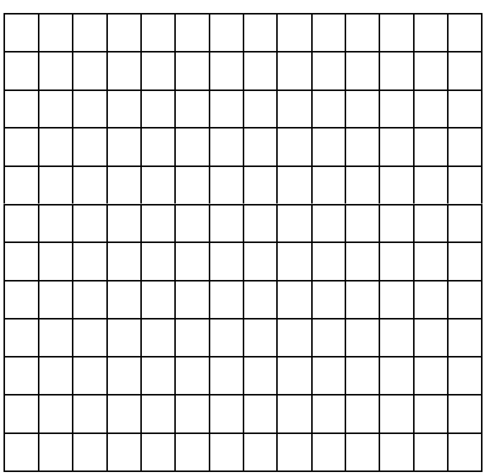
$y^2 = x - 1$

Since the equations are the same, the graph will be symmetric about the x-axis.

A graph is symmetric with respect to the origin if for all points $(x\ ,\ y)$ on the graph there are also the points $(-x\ ,\ -y)$. For example, if the point $(3\ ,\ -2)$ is on the graph, then the point $(-3\ ,\ 2)$ is also on the graph. Algebraically, the substitution of –x for x **and** $-y$ for y leads to the same equation. A function that is symmetric about the origin is called an **odd** function.

Example:

Graphically



Algebraically:

$y = x^3$

To show the graph of the above equation would be symmetric about the x-axis, plug in –x for x and –y for y and simplify.

$(-y) = (-x)^3$

$-y = -x^3$

$y = x^3$

Since the equations are the same, the graph will be symmetric about the origin.

Sketch a graph of the equation and label the x- and y-intercepts. List the type of equation, describe the shape of the graph, and list its characteristics.

1. $y = 3x + 2$

Type of equation:
Shape:
Characteristics:

2. $y = -x^2 + 3$

Type of equation:
Shape:
Characteristics:

3. $\frac{1}{2}y^2 = x$

Type of equation:
Shape:
Characteristics:

4. $y = \sqrt{x} + 1$

Type of equation:
Shape:
Characteristics:

5. $y = x^3 - 2$

Type of equation:
Shape:
Characteristics:

6. $x^2 + y^2 = 9$

Type of equation:
Shape:
Characteristics:

Standard Equation of a Circle

The circle with a center of $(h\ ,\ k)$ and radius r is given by the equation: $(x-h)^2 + (y-k)^2 = r^2$

Equations of Semicircles of a circle $x^2 + y^2 = r^2$

The upper half is given by $y = \sqrt{r^2 - x^2}$. The lower half is given by $y = -\sqrt{r^2 - x^2}$.

The right half is given by $x = \sqrt{r^2 - y^2}$. The left half is given by $x = -\sqrt{r^2 - y^2}$.

Sketch the graph of the circle or semicircle.

7. $(x+3)^2 + (y-2)^2 = 16$

8. $y = \sqrt{4 - x^2}$

Find an equation of the circle that satisfies the stated conditions.

9. Center $C(-4\ ,1)$ radius 3

10. Center at the origin passing through $P(4\ ,-7)$

11. Center $C(4\ ,-1)$ tangent to the x-axis

12. Tangent to both axes, center in the fourth quadrant, radius 3

13. Endpoints of a diameter $A(-5\ ,2)$ and $B(3\ ,6)$

Find the center and radius of the circle with the given conditions.

14. $x^2 + y^2 + 8x - 10y + 37 = 0$

15. $9x^2 + 9y^2 + 12x - 6y + 4 = 0$