Math 4 Pre-Calculus
Lecture 5, Part 1

Homework:
pg. 219, 221: 5–25odd, 27–71eoo, 73–85odd
pg 227, 228: 9–25odd

Functions — 3.1 and 3.2

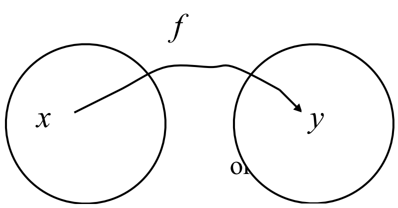
Correspondence

A correspondence is a relationship between two sets. A correspondence or one set mapping to another may be denoted by an arrow "→".
Examples of correspondences:
A set of students at Cabrillo → a set of student ID numbers
A set of chairs in the classroom → a set of Cabrillo students
A set of numbers denoted by x → a set of numbers denoted by y

Functions

A function, f, is a correspondence between two sets in which each element in the first set (x) is assigned to only one element in the second set (y). In other words, for every input (x), there is **only one** corresponding output (y).
Are any of the above correspondences a function?

Set Notation of Functions



D — *Domain* *"first set"* *"inputs"* *argument*

E — *Range* *"second set"* *"outputs"*

$f : D \to E$ $\qquad$ $D \xrightarrow{f} E$

Domain

The domain of a function is the set of all x-values (elements in the first set of the correspondence).
The domain is the set of values on which the function is defined.

$$\{x \mid f(x) \text{ is defined}\}$$

Range

The range of a function is the set of all y-values (elements in the second set of the correspondence).
The range is the set of all values that the function can calculate.

$$\{y \mid y = f(x) \text{ for some value } x \text{ in the domain}\}$$

Function Notation

A function takes an input and gives an output.
A function takes an input = an output.
A function (input) = an output.
f(input) = output.
f(x) = output.

Phrases that indicate Functions:

y is expressed in terms of x
y is in terms of x
y depends on x
y is caused by x
y is mapped by x
Can you think of any others? Let me know...

Example:
A function takes an input and gives its square. In function notation: $\quad f(\text{input}) = \text{input}^2 \quad$ or $\quad f(x) = x^2$

Evaluate.

1. If $f(x) = -x^2 + 4x - 1$, find $f(3)$, $f(0)$ and $f\left(-\frac{1}{2}\right)$.

If a and h are real numbers, find: *a)* $f(a)$ *b)* $f(-a)$ *c)* $-f(a)$ *d)* $f(a+h)$ *e)* $f(a)+f(h)$

f) $\dfrac{f(a+h)-f(a)}{h}$, if $h \neq 0$.

2. $f(x) = 3 - x^2$

a)

b)

c)

d)

e)

f)

If a and h are real numbers, find: *a)* $\dfrac{g(a+h)-g(a)}{h}$, if $h \neq 0$ *b)* $g\left(\dfrac{1}{a}\right)$ *c)* $\dfrac{1}{g(a)}$

d) $g\left(\sqrt{a}\right)$ *e)* $\sqrt{g(a)}$.

3. $g(x) = \dfrac{x^2}{x+1}$

a)

b)

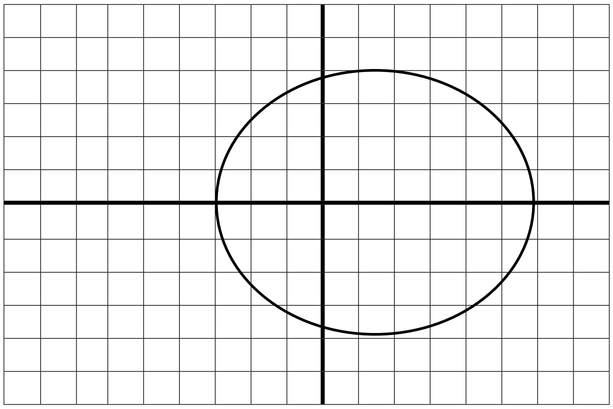
c)

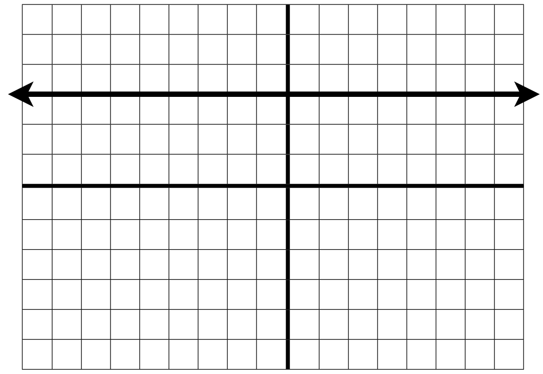
d)

e)

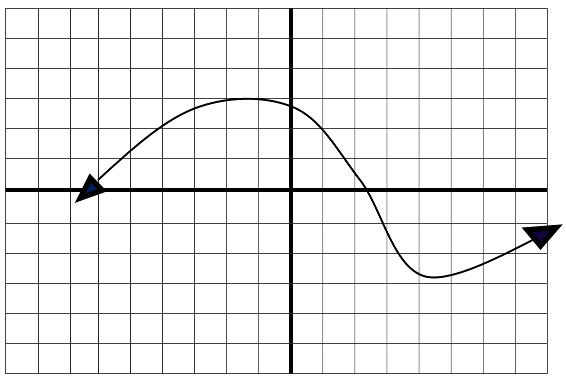
Vertical Line Test

The graph of a set of points, $(x\ ,\ y)$, in a coordinate plane is the graph of a function if every (and any) vertical line intersects the graph in at most one point.   *Explain why the graph is or is not the graph of a function.*





Function Notation



a) For the graph of the function f sketched in the figure determine *the domain*

b) *the range*

c) $f(2)$

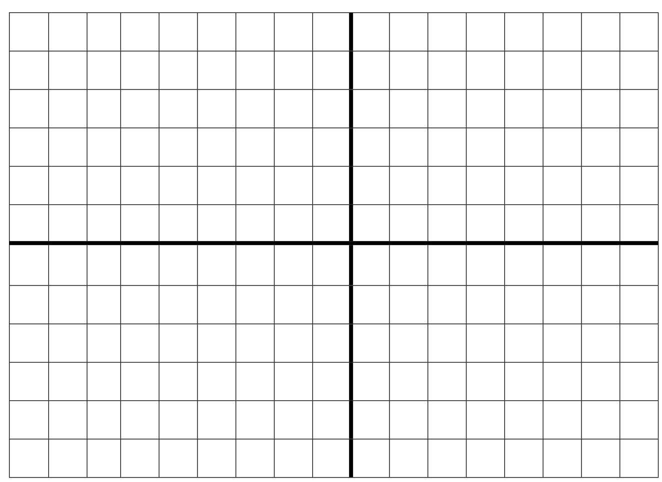
d) *all x such that* $f(x) = 2$

e) *all x such that* $f(x) > 2$

f) *find the intervals on which* f *is increasing*

g) *find the intervals on which* f *is decreasing*

h) *find the intervals on which* f *is constant*



Finding the Domain

To find the domain of a function given the equation:
1.Assume that the domain is all real numbers.
2.Take out numbers (or intervals of numbers) for which the function is undefined:

a) Take out x's that make a denominator zero. Set denominator $= 0$ to find numbers to take out.

b) Take out x's that cause imaginary numbers (even roots of negative radicands). (Set radicand ≥ 0 to find x's that work.

c) Take out x's for which the argument of a logarithm is negative. (This will be covered in Chapter 5).

Find the domain of each function.

4. $f(x) = x^3 + 10$

5. $g(x) = \sqrt{x - 5}$

6. $h(x) = \dfrac{5}{x + 2}$

7. $f(x) = \dfrac{\sqrt{1 - x}}{x^2 + 2x - 15}$

8. $k(x) = \sqrt{x + 3} + \sqrt{3 - x}$

9. $p(x) = \sqrt{x^2 + 6x + 8}$

Simplify the difference quotient: $\dfrac{f(x+h) - f(x)}{h}$, if $h \neq 0$

10. $f(x) = -x^2 + 2x$