

Operations on Functions — 3.1, 3.6, and 6.1

Definitions

Terminology	Function Notation
Sum of two functions $f + g$	$(f + g)(x) = f(x) + g(x)$
Difference of two functions $f - g$	$(f - g)(x) = f(x) - g(x)$
Product of two functions fg	$(fg)(x) = f(x) \times g(x)$
Difference of two functions $\frac{f}{g}$	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad g(x) \neq 0$

Find $(f + g)(2)$ and $(fg)(2)$.

1. $f(x) = 3x^2$, $g(x) = \frac{1}{x}$

Find **a)** $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$ **b)** the domain of $f + g$, $f - g$, and fg , **and**

c) the domain of $\frac{f}{g}$.

2. $f(x) = \sqrt{4 - x^2}$ $g(x) = 2x + 3$

3. A meteorologist is inflating a spherical balloon with helium gas. If the radius of the balloon is changing at a rate of 1.5 cm/sec, express the volume V of the balloon as a function of time t (in seconds).
- a) First express *the radius of the balloon as a function of time*.
- b) Next express *the volume as a function of the radius*.
- c) Now, express *the volume as a function of time*; this function is a composition of the two previous functions.

3.5. Do 3 sample problems from page 267, 268

Definition

The **composite function** $f \circ g$ of two functions f and g is defined by $(f \circ g)(x) = f(g(x))$.

The **domain** of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f . To find the domain of $f \circ g$, find the intersection of the domain of g and the domain of the simplified outcome of $f \circ g$.

Find **a)** $(f \circ g)(x)$, $(g \circ f)(x)$, $f(g(-2))$ and $g(f(3))$.

4. $f(x) = -x - 1$ $g(x) = 4x^2$

Find $(f \circ g)(x)$ and the domain of $(f \circ g)(x)$, and $(g \circ f)(x)$ and the domain of $(g \circ f)(x)$.

5. $f(x) = \sqrt{x-4}$ $g(x) = 2x^2$

6. $f(x) = \frac{1}{x-1}$ $g(x) = x-1$

Several values of two functions f and g are listed in the following tables.

x	0	1	2	3	4
$f(x)$	2	3	1	0	5

x	0	1	2	3	4
$g(x)$	1	0	3	2	5

If possible, find

a) $(f \circ g)(1)$ b) $(g \circ f)(1)$

c) $(f \circ f)(1)$ d) $(g \circ g)(1)$

e) $(f \circ g)(4)$

Find a composite function form for y . There is more than one possible answer.

8. $y = \sqrt[4]{x^3 - 16}$

9. $y = \frac{1}{\sqrt{|x| + 5}}$