Math 4 Pre-Calculus
Lecture 7, Part1

Homework:
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Inverse Functions — 6.2

Definition of a One-to-One Function

A function f with domain D and range R is a **one-to-one function** if either of the following equivalent conditions is satisfied:

1. Whenever $a \neq b$ in D, then $f(a) \neq f(b)$ in R.
2. Whenever $f(a) = f(b)$ in R, then $a = b$ in D.

Determine whether a function f is one-to-one.

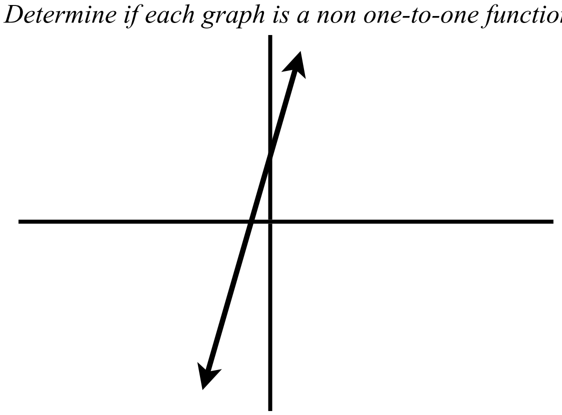
1. $f(x) = -2x + 1$

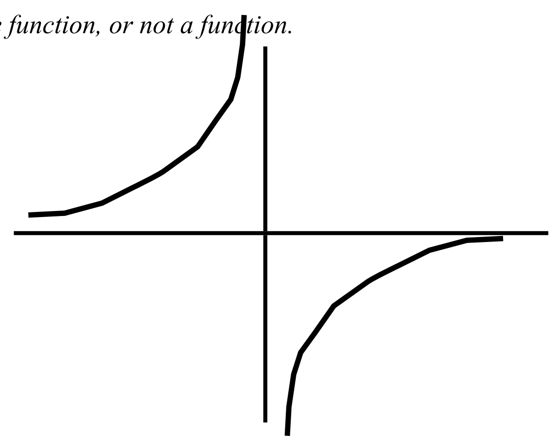
2. $f(x) = (x+2)^2 - 3$

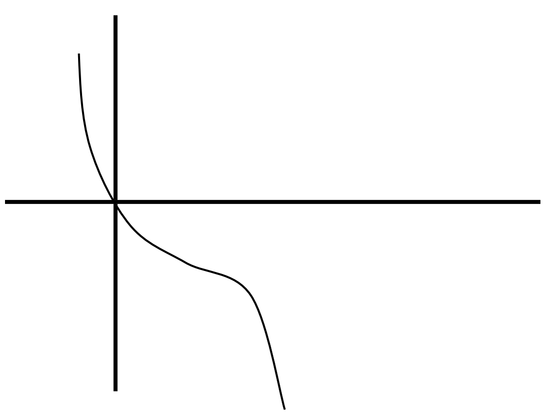
Horizontal Line Test

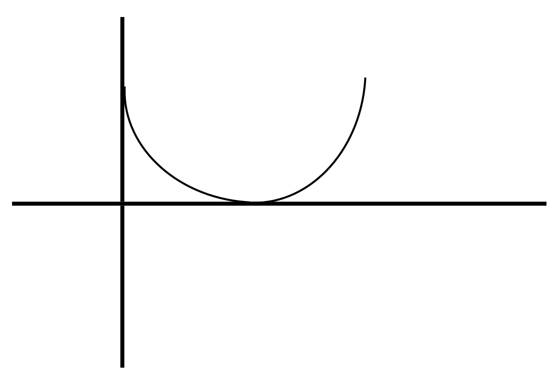
A function f is one-to-one if and only if every **horizontal** line intersects the graph of f in at most one point.

Determine if each graph is a non one-to-one function, a one-to-one function, or not a function.









Inverse Functions

Let f be a one-to-one function with domain D and range R. A function g with domain R and range D is the inverse function of f provided the following condition is true for every x in D and every y in R.

$$y = f(x) \text{ if and only if } x = g(y).$$

In other words, the domain and range switch, and g maps every y (output of f) back to its original x (input of f).

All one-to-one functions have inverses.

Theorem on Inverse Functions

Let f be a one-to-one function with domain D and range R. If g is a function with domain R and range D, then g is the inverse function f if and only if both of the following conditions are true.

1. $g(f(x)) = x$ for every x in D.
2. $f(g(y)) = y$ for every y in R.

Notation

The inverse of a one-to-one function f is denoted by f^{-1}. This means if f and f^{-1} are inverses, the following is true:

1. $f^{-1}(f(x)) = x$ for every x in the domain of f.
2. $f(f^{-1}(x)) = x$ for every x in the domain of f^{-1}.

Domain and Range

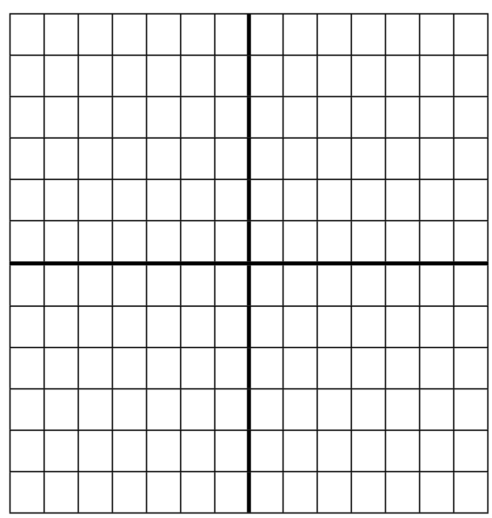
If f and f^{-1} are inverses, then

1. The domain of f is equal to the range of f^{-1}.
2. The domain of f^{-1} is equal to the range of f.

Graphs of Inverses

If f and f^{-1} are inverses, then the graphs of f and f^{-1} are reflections of each other about the line $y = x$, or are symmetric with respect to this line. In other words if the point (a, b) is on the graph of f, then the point (b, a) is on the graph of f^{-1}.

Use the theorem on inverse functions to prove that f and g are inverse functions of each other, and sketch graphs of f and g on the same coordinate plane.



3. $f(x) = x^3 - 1$ $\qquad$ $g(x) = \sqrt[3]{x+1}$

Determine the domain and range of f^{-1} *for the given function without actually finding* f^{-1}.

4. $f(x) = x^2 - 2, \quad x \geq 0$

Find the inverse of the function f .

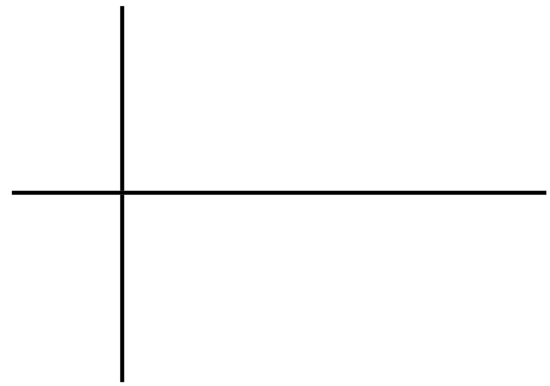
5. $f(x) = x^2 - 2, \quad x \geq 0$

6. $f(x) = \dfrac{4x}{x - 2}$

7. $f(x) = -\sqrt{x + 5} - 2$

Let $h(x) = 3 - x$. *Use* h*, the table, and the graph to evaluate each expression.*

x	0	1	2	3	4
$f(x)$	2	3	1	0	5



a) $(h \circ g)(2)$

b) $(f^{-1} \circ h)(0)$

c) $(f^{-1} \circ g^{-1})(-2)$

8. *The graph of a one-to-one function* f *is shown. Use the reflection property to sketch the graph of* f^{-1}. *Find the domain and range of the function. Find the domain and range of the inverse function.*