Homework:
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Math 4 Pre-Calculus
Lecture 7, Part 2

Transformations — 3.5

Vertical Translations (Up or Down)

Function	Basic Function	Transformation Notation	Algebraic Change	Basic Function Movement	Graph
$y = x^2 + 2$					
$y = \sqrt{x} - 3$					

If f is a function, and c is a positive constant, then

1. $y = f(x) + c$ is the graph of $y = f(x)$ shifted up (vertically) c units.
2. $y = f(x) - c$ is the graph of $y = f(x)$ shifted down (vertically) c units.

Horizontal Translations (Left or Right)

Function	Basic Function	Transformation Notation	Algebraic Change	Basic Function Movement	Graph
$y = (x-1)^2$					
$y = \sqrt{x+2}$					

If f is a function, and c is a positive constant, then

1. $y = f(x + c)$ is the graph of $y = f(x)$ shifted to the **LEFT** (horizontally) c units.
2. $y = f(x - c)$ is the graph of $y = f(x)$ shifted to the **RIGHT** (horizontally) c units.

Vertical Compressions or Stretches

Function	Basic Function	Transformation Notation	Algebraic Change	Basic Function Movement	Graph
$y = 3\|x\|$					
$y = \frac{1}{4}\sqrt{x}$					

If f is a function, the graph of $y = cf(x)$

1. is the graph of $y = f(x)$ stretched (vertically) by a factor of c, if $c > 1$.
2. is the graph of $y = f(x)$ compressed (vertically) by a factor of c, if $0 < c < 1$.

Horizontal Compressions or Stretches

Function	Basic Function	Transformation Notation	Algebraic Change	Basic Function Movement	Graph
$y = \sqrt{3x}$					
$y = \left\|\frac{1}{2}x\right\|$					

If f is a function, the graph of $y = cf(x)$ multiplies the horizontal distance between a point of the graph of $y = f(x)$ and the y-axis by a factor of $\frac{1}{c}$.

1. If $c > 1$, this results in a horizontal COMPRESSION of $y = f(x)$.
2. If $0 < c < 1$, this results in a horizontal STRETCH of $y = f(x)$.

Reflections (Across the x-axis and the y-axis)

Function	Basic Function	Transformation Notation	Algebraic Change	Basic Function Movement	Graph
$y = -\lvert x\rvert$					
$y = \sqrt{-x}$					

If f is a function, then

1. $y = -f(x)$ is the graph of $y = f(x)$ reflected vertically across the x-axis. The entire graph of $y = f(x)$ flips upside down.
2. $y = f(-x)$ is the graph of $y = f(x)$ reflected horizontally across the y-axis. The entire graph of $y = f(x)$ flips over itself sideways.

Absolute Value Effects

Function	Basic Function	Transformation Notation	Algebraic Change	Basic Function Movement	Graph
$y = \lvert x^3\rvert$					
$y = \sqrt{\lvert x\rvert}$					

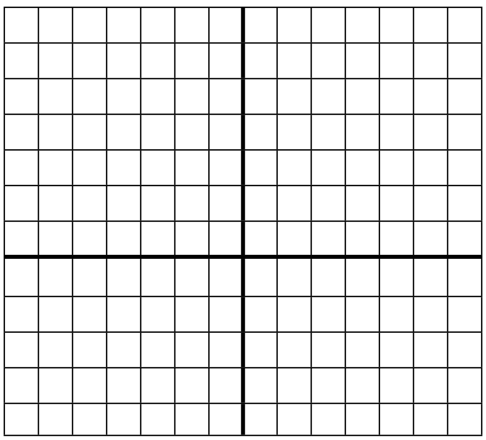
If f is a function, then

1. $y = |f(x)|$ makes all points with a negative y-value for $y = f(x)$ reflect vertically up across the x-axis. The new transformation only appears in Quadrants I and II.

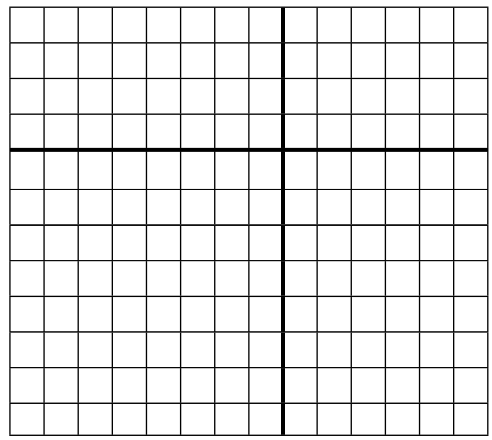
2. $y = f(|x|)$ is the same as the graph of $y = f(x)$ for Quadrants I and IV (where x > 0) in addition to that reflected horizontally across the y-axis. The new transformation is an even function.

Use the correct tool-kit graph and transformation knowledge to graph each transformation of f *on the same coordinate axes.*

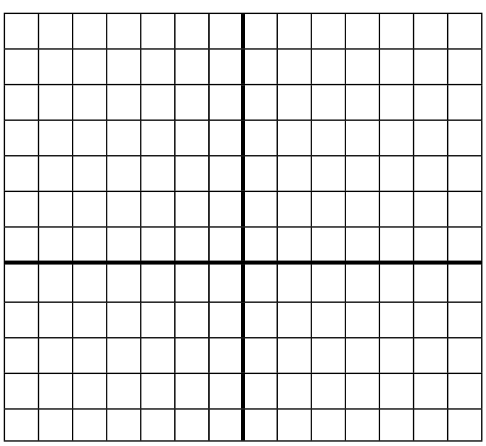
1. Graph $f(x) = (x - c)^2$ for $c = 1, -2, 4$



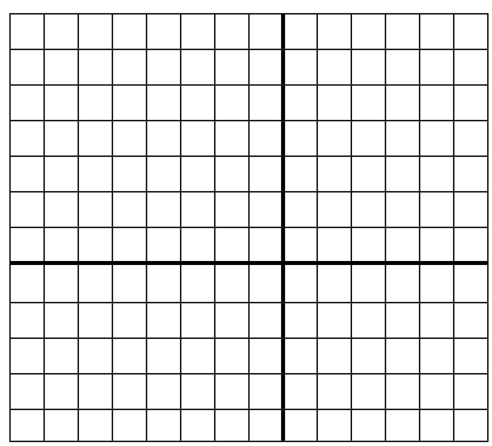
2. Graph $f(x) = \sqrt{x} + c$ for $c = -1, \frac{1}{2}, -3$



3. Graph $f(x) = (cx)^3$ for $c = 2, \frac{1}{2}, -2$



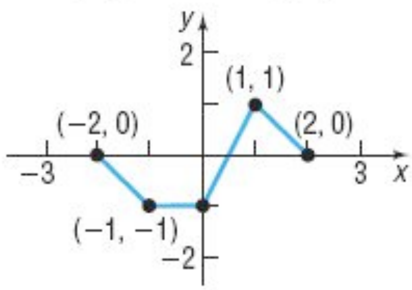
4. Graph $f(x) = c\frac{1}{x}$ for $c = 2, \frac{1}{2}, -2$



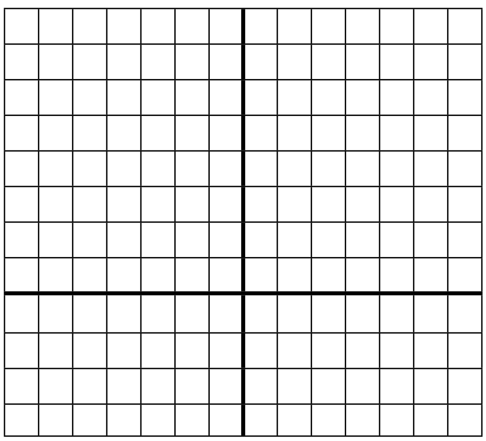
Combinations of Transformations

Vertical transformations can be done before or after horizontal transformations.
For vertical (y) transformations, first stretch/compress/flip vertically, then shift up or down.
For horizontal (x) transformations, first factor out the stretch/compression, then stretch/compress/flip horizontally, and then shift left or right.

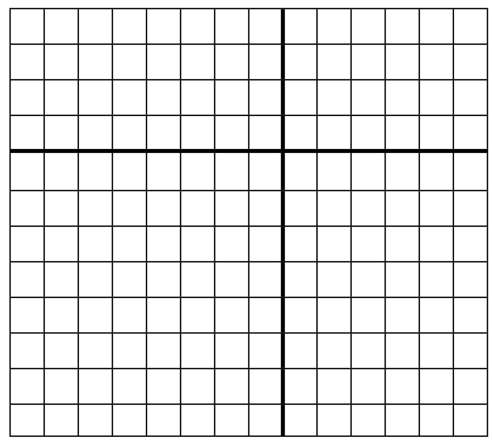
Use the graph below of the function f to graph each transformation.



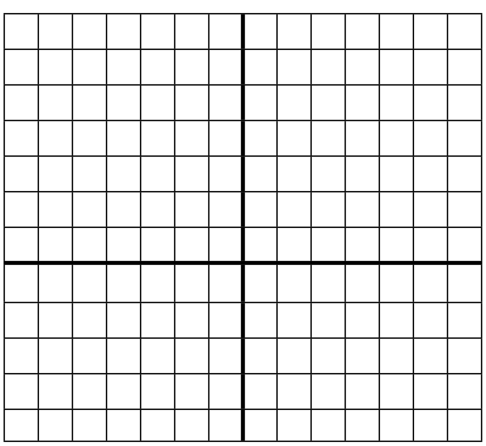
5. Graph $y = -f(x-1)+2$



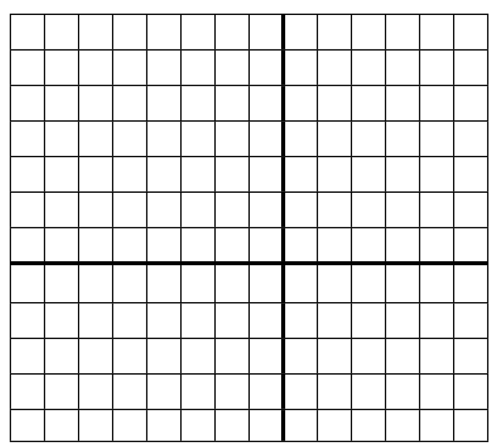
6. Graph $y = f(x+3)-5$



7. Graph $y = 2f(|x|)$



8. Graph $y = |f(x+3)|$



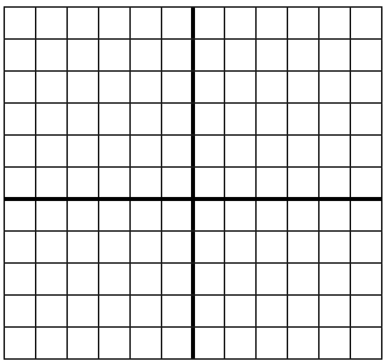
Piecewise Defined Functions

A piece-wise defined function is a function that has different “rules” for various x-values.

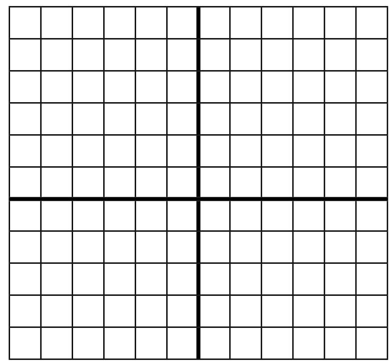
Graph:

$$f(x) = \begin{cases} (x+2)^2 - 1, & x < -2 \\ 3, & -2 \le x < 0 \\ \frac{1}{2}x + 3, & x \ge 0 \end{cases}$$

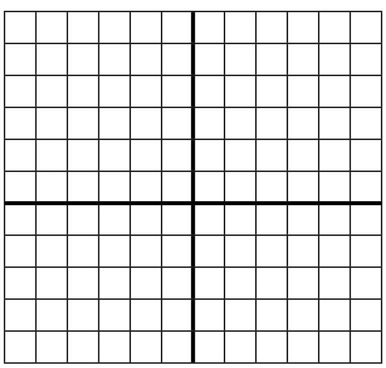
First graph $f(x) = (x+2)^2 - 1$



Next graph $f(x) = 3$



Now graph $f(x) = \frac{1}{2}x + 3$



Finally graph $f(x) = \begin{cases} (x+2)^2 - 1, & x < -2 \\ 3, & -2 \le x < 0 \\ \frac{1}{2}x + 3, & x \ge 0 \end{cases}$