

Quadratic Polynomial Functions — 4.3 and 4.4

Basic Quadratic Function $f(x) = ax^2 + bx + c$ where $a = 1$ and $b = 0$ and $c = 0$

$$f(x) = x^2$$

Domain:

Range:

Vertex:

Vertical Axis:

$a = 1$, so the graph is concave up and the y-value of the vertex is a minimum function value.

x-intercept(s) or root(s) or zero(s):

y-intercept:

Vertex Form of a Quadratic Function

$$f(x) = a(x - h)^2 + k, \quad a \neq 0$$

Domain:

Vertex:

If $a > 0$, the graph is concave up and the y-value of the vertex is a **minimum** function value. Therefore the **range** is $[k, \infty)$.

If $a < 0$, the graph is concave down and the y-value of the vertex is a **maximum** function value. Therefore the **range** is $(-\infty, k]$.

x-intercept(s) or root(s) or zero(s): Find by setting $y = 0$ and solving for x. The x-intercept(s) is(are) a point(s): $(?, 0)$

y-intercept: Plug in 0 for x to find y. In other words, find $f(0)$. The y-intercept is a point: $(0, ?)$

Standard Form of a Quadratic Function

$$f(x) = ax^2 + bx + c, \quad a \neq 0$$

Domain:

Vertex: The x-value of the vertex is . The y-value of the vertex is .

If $a > 0$, the graph is concave up and the y-value of the vertex is a **minimum** function value.

If $a < 0$, the graph is concave down and the y-value of the vertex is a **maximum** function value.

x-intercept(s) or root(s) or zero(s): Find by setting $y = 0$ and factoring or using the quadratic formula to solve for x. The x-intercept(s) is(are) a point(s): $(?, 0)$

y-intercept: Plug in 0 for x to find y. In other words, find $f(0)$. The y-intercept is a point: $(0, ?)$

Factored Form of a Quadratic Function

$$f(x) = a(x - x_1)(x - x_2), \quad a \neq 0$$

Domain:

Vertex: To find the vertex, write in general form or vertex form.

If $a > 0$, the graph is concave up and the y-value of the vertex is a **minimum** function value.

If $a < 0$, the graph is concave down and the y-value of the vertex is a **maximum** function value.

x-intercept(s) or root(s) or zero(s): The x-intercepts are the points: $(x_1, 0)$ and $(x_2, 0)$.

y-intercept: Plug in 0 for x to find y. In other words, find $f(0)$. The y-intercept is a point: $(0, ?)$

1. Graph the equation.

$$f(x) = -2(x + 3)^2 + 5,$$

Domain:

Vertex:

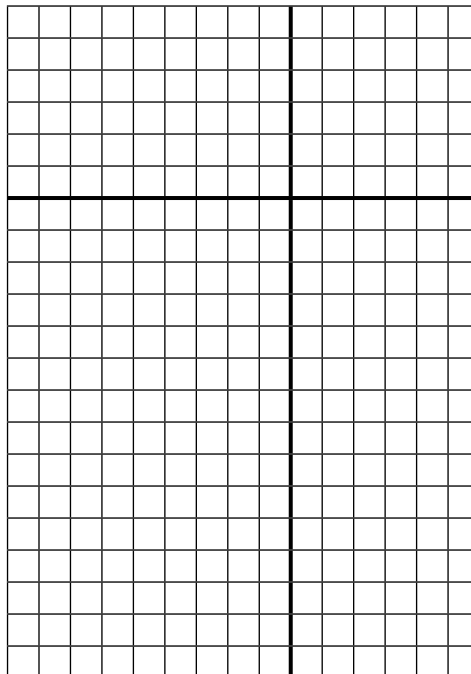
Vertical Axis:

Value of "a":

Range:

x-intercept(s) or root(s) or zero(s):

y-intercept:



2. Write the quadratic function in vertex form, and then graph the function.

$$f(x) = x^2 - 6x + 11$$

Domain:

Vertex:

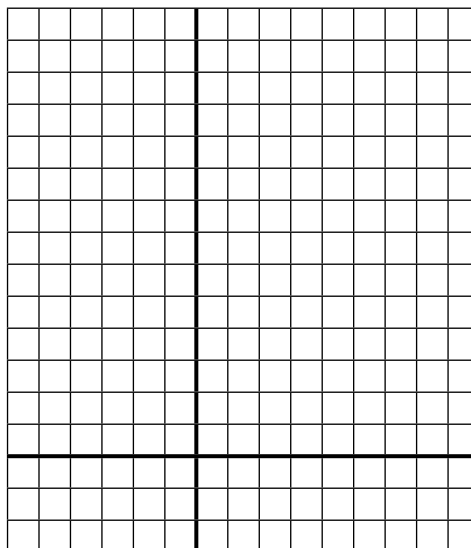
Vertical Axis:

Value of "a":

Range:

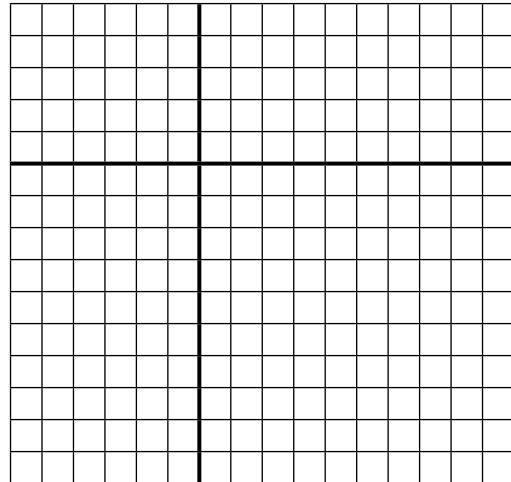
x-intercept(s) or root(s) or zero(s):

y-intercept:



3. Write the quadratic function in vertex form, and then graph the function.

$$f(x) = -\frac{1}{2}x^2 + 4x - 7$$



Domain:

Vertex:

Vertical Axis:

Value of “a”:

Range:

x-intercept(s) or root(s) or zero(s):

y-intercept:

4. Write the quadratic function in vertex form.

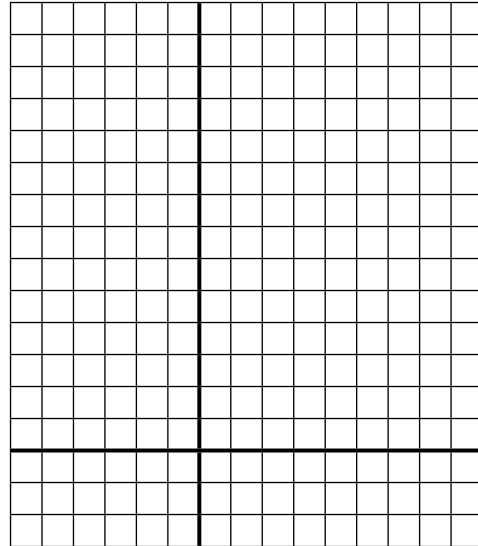
$$f(x) = ax^2 + bx + c$$

Finding the Vertex of a Quadratic Function:

To find the vertex of a quadratic function, you may either:

1. Complete the square to get the function in vertex form.
2. Write the function in standard form. The vertex will be $(-\frac{b}{2a}, f(-\frac{b}{2a}))$. In other words, the x-value of the vertex is $-\frac{b}{2a}$ and you can find the y-value of the vertex by plugging the x-value into the function.

Graph the function. $f(x) = 2x^2 - 6x + 4$



Domain:

Vertex:

Vertical Axis:

Range:

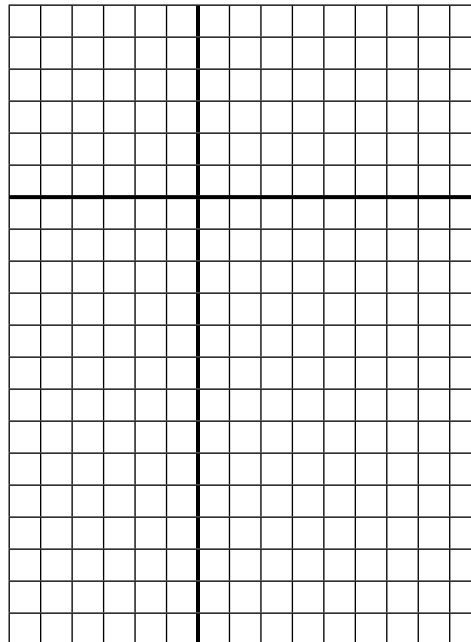
Value of “a”:

x-intercept(s) or root(s) or zero(s):

y-intercept:

5. Graph the function.

$$f(x) = 3(x + 2)(x - 4)$$



Domain:

Vertex:

Vertical Axis:

Value of “a”:

Range:

x-intercept(s) or root(s) or zero(s):

y-intercept:

6. Find the standard equation of a parabola that has a vertical axis and satisfies the given condition.

Vertex (3, 5), passing through (4, 1)

7. Find the standard equation of a parabola that has a vertical axis and satisfies the given condition.

x-intercepts $(2, 0)$ and $(-1, 0)$, passing through $(0, -3)$

8. An object is projected vertically upward with an initial velocity of v_0 ft/sec, and its distance $s(t)$ in feet above the ground after t seconds is given by the formula $s(t) = -16t^2 + v_0t$.

a) If the object hits the ground after 12 seconds, find its initial velocity v_0 .

b) Find its **maximum** distance above the ground.

9. A farmer wishes to put a fence around a rectangular field and then divide the field into three rectangular plots by placing two fences parallel to one of the sides. If the farmer can afford only 1000 yards of fencing, what dimensions will give the **maximum** rectangular area?