

Polynomial Functions — 5.1

Polynomial Function

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

Domain:

The **range** depends on the global behavior which can be determined by the degree of the polynomial and the sign of the leading coefficient.

y-intercept: Plug in 0 for x to find y. In other words, find $f(0)$. The y-intercept is a point: $(0, ?)$.

x-intercept(s) or root(s) or zero(s): Find by setting $y = 0$ and solving for x. The x-intercept(s) is(are) a point(s): $(?, 0)$.

Roots of polynomial functions will be discussed through Chapter 4.

Polynomial Functions with an ODD degree

Domain:

Global Behavior:

If $a > 0$, the graph is a “disco shape : right side up”. **Range:**

In mathematical terms, this means: as $x \rightarrow \infty$, $f(x) \rightarrow \infty$ and as $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$

If $a < 0$, the graph is a “disco shape : left side up”. **Range:**

In mathematical terms, this means: as $x \rightarrow \infty$, $f(x) \rightarrow -\infty$ and as $x \rightarrow -\infty$, $f(x) \rightarrow \infty$

Polynomial Functions with an EVEN degree

Domain:

Global Behavior:

If $a > 0$, the graph is a “touchdown shape”. **Range:**

In mathematical terms, this means: as $x \rightarrow \infty$, $f(x) \rightarrow \infty$ and as $x \rightarrow -\infty$, $f(x) \rightarrow \infty$

If $a < 0$, the graph is a “
shape”. **Range:**

In mathematical terms, this means: as $x \rightarrow \infty$, $f(x) \rightarrow -\infty$ and as $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$

1. Graph the equation. $f(x) = 9x - x^3$

Domain:

Degree:

Value of a:

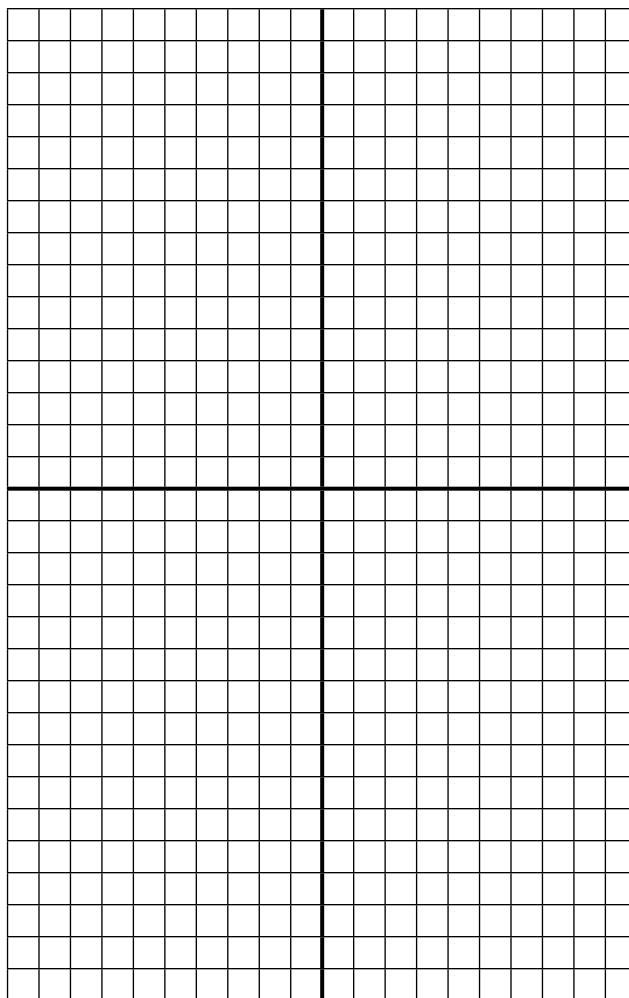
Global Behavior:

Range:

Symmetry:

y-intercept:

x-intercept(s) or root(s) or zero(s):



Sign chart:

2. Graph the equation. $f(x) = -x^4 + 12x^2 - 27$

Domain:

Degree:

Value of a:

Global Behavior:

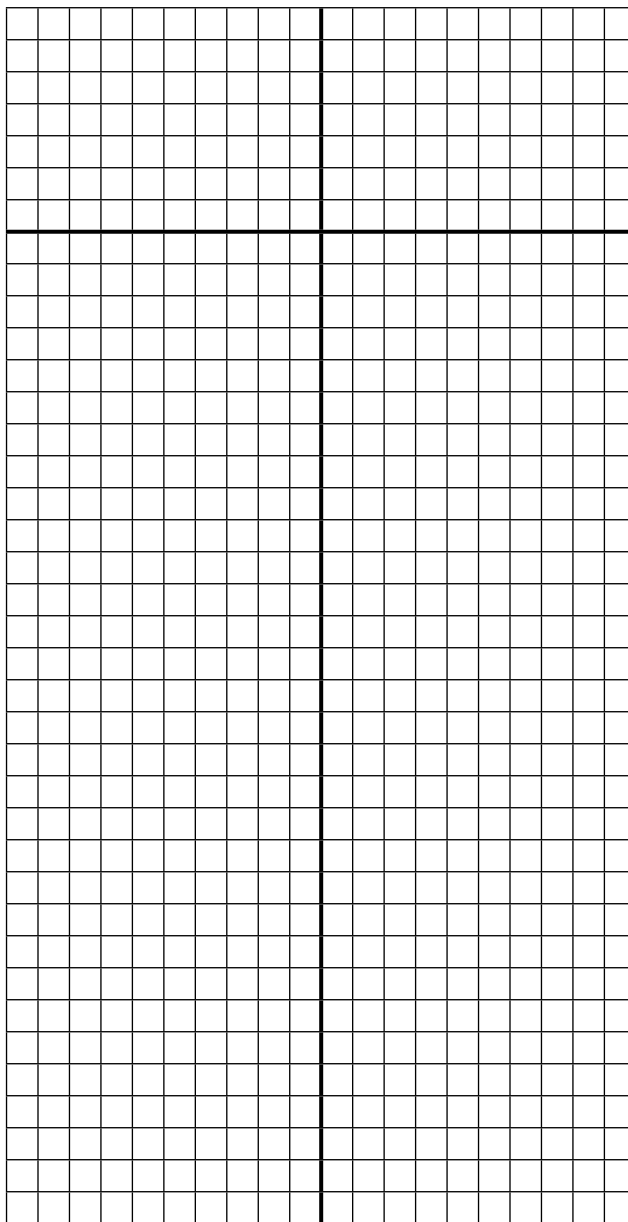
Range:

Symmetry:

y-intercept:

x-intercept(s) or root(s) or zero(s):

Sign chart:



3. Graph the equation. $f(x) = x^2(x+2)^2(x-1)$

Domain:

Degree:

Value of a:

Global Behavior:

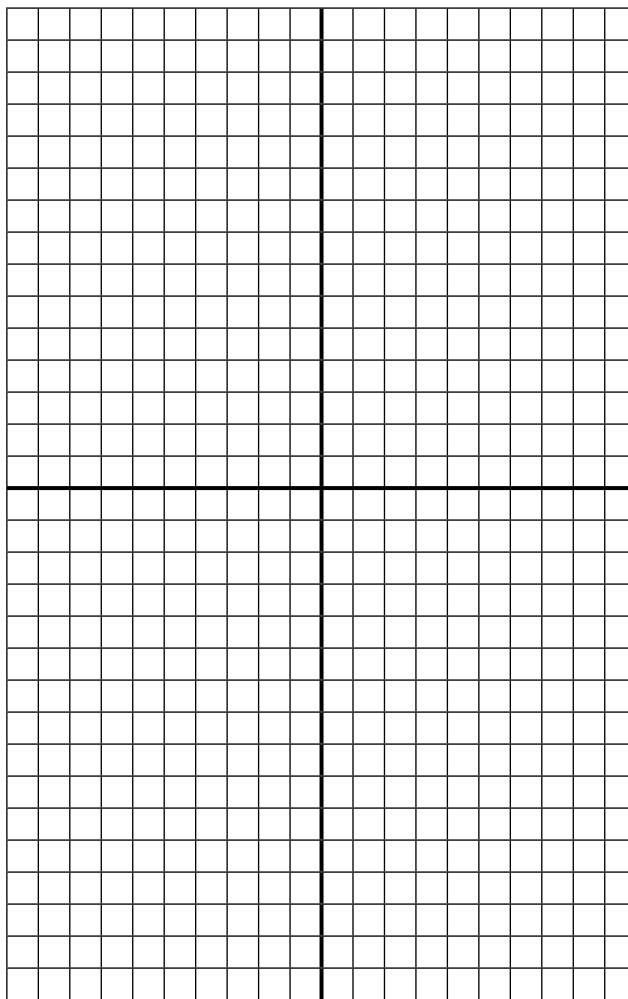
Range:

Symmetry:

y-intercept:

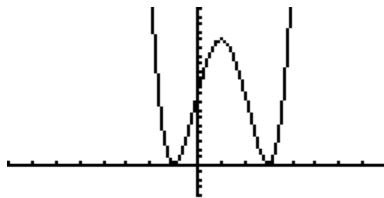
x-intercept(s) or root(s) or zero(s):

Sign chart:



4. Match each graph with an equation. Scales on all graphs: $X_{scl}=1$ and $Y_{scl}=1$.

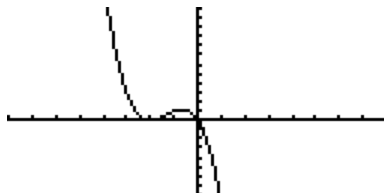
a) $f(x) = x^2(x+2)^2$



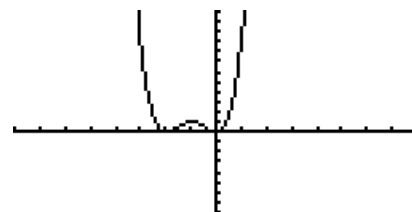
b) $f(x) = -x(x+2)^2$



c) $f(x) = (x+1)^2(x-3)^2$

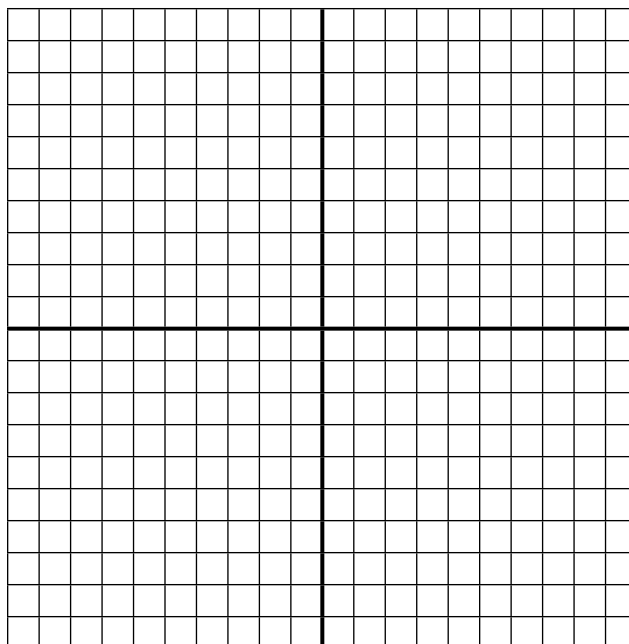


d) $f(x) = (x+1)^2(x-3)$



5. Sketch the graph of a polynomial given the sign diagram.

Sign of $f(x)$	+	+	+
	-4	0	2



Intermediate Value Theorem for Polynomial Functions

If f is a polynomial function, and $f(a) \neq f(b)$ for $a < b$, then f takes on every value between $f(a)$ and $f(b)$ in the interval $[a, b]$.

Right now, we are using this theorem to prove the existence of a zero (x-intercept) between any two given x-values in which one has a positive y-value and the other has a negative y-value.

6. Use the intermediate value theorem to show that f has a zero between a and b .

$$f(x) = 2x^3 + 5x^2 - 3 \qquad a = -3, \quad b = -2$$