Homework
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Properties of Division — R.4 and R.6

Review of Polynomial Functions

The following two functions are the same:

$$f(x) = x^3 + 4x^2 - 7x - 10 \qquad f(x) = (x+1)(x-2)(x+5)$$

The first one is multiplied out,; the second is in factored form. Polynomials that are written as a product of linear and quadratic factors are much easier to graph then those that are multiplied out, because the zeros (roots or x-intercepts) can be readily seen.

The next few sections (4.2–4.4) give tools for finding the zeros (roots or x-intercepts) of polynomial functions.

Tools for finding zeros of a polynomial function:

1. Factoring
2. Quadratic Formula
3. Polynomial Long Division or Synthetic Division
4. Theorems:
   - Remainder and Factor Theorems
   - Fundamental Theorem of Algebra (& more)
   - Conjugate Pair Zeros
   - Product of Linear and Quadratic Factors
   - Rational Zeros of a Polynomial

Polynomial Long Division

Find the quotient and remainder if $f(x)$ is divided by $p(x)$. *Use this information to write* $f(x)$ *as a product of linear and quadratic factors irreducible over* $\mathbb{R}$.

1. $f(x) = -2x^3 - 8x^2 - 2x + 12 \qquad p(x) = x - 1$

Division Algorithm for Polynomials

If $f(x)$ and $p(x)$ are polynomials and $p(x) \neq 0$ then there exists unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = p(x) \times q(x) + r(x) \quad \text{or} \quad f(x) = p(x) \times \text{quotient} + \text{remainder},$$

where the degree of $r(x)$ is less than the degree of $p(x)$.

Synthetic Division

Synthetic division only works when dividing a polynomial by a linear factor.

Find the quotient and remainder if $f(x)$ is divided by $p(x)$.

2. $f(x) = x^4 + 3x^2 + 6x - 5$ $\qquad$ $p(x) = x - 2$

3. *Find* $f(2)$ *for* $f(x) = x^4 + 3x^2 + 6x - 5$

Remainder Theorem

If a polynomial $f(x)$ is divided by $x - c$ (linear factor), then the remainder is $f(c)$.

Factor Theorem

A polynomial $f(x)$ has a linear factor $x - c$, if and only if $f(c) = 0$.

4. Show that $x-2$ is a factor of $f(x)=x^3-4x^2+3x+2$

Find the quotient and remainder if $f(x)$ is divided by $p(x)$. *Use this information to write* $f(x)$ *as a product of linear and quadratic factors irreducible over* $\mathbb{R}$.

5. $f(x)=x^6-64$ $p(x)=x^2-2x+4$

Use synthetic division to find $f(c)$.

6. $f(x) = x^3 - 3x^2 - 8$ $c = 1 + \sqrt{2}$

Write a polynomial $f(x)$ *with a leading coefficient of 1 and having the given degree and zeros.*

7. degree 4; zeros ±2, 0, 1