Math 4 Pre-Calculus
Lecture 9 Part 2

**Homework**
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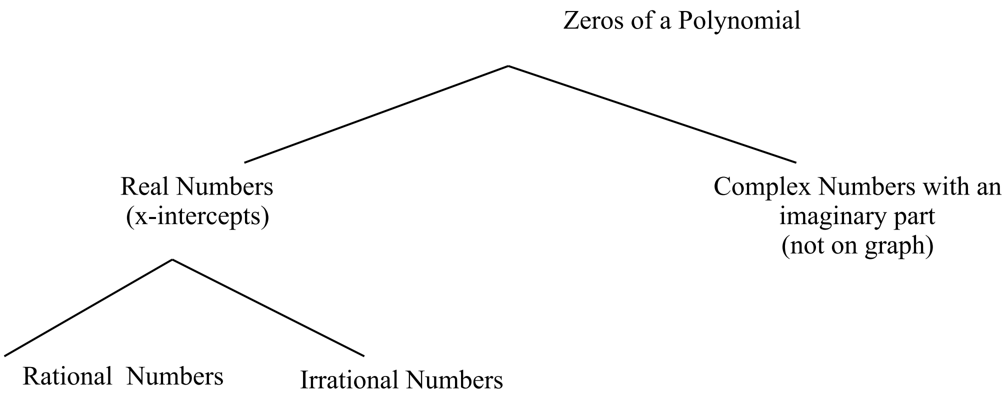
# Zeros of Polynomials — 5.5 and 5.6




## Fundamental Theorem of Algebra

If a polynomial $f(x)$ has positive degree and complex (could be all real) coefficients then $f(x)$ has at least one complex (could be real) zero.

The Fundamental Theorem of Algebra tells us that every polynomial can be written as a product of complex linear factors.

*Find all zeros. Write $f(x)$ as a product of complex zeros.*

1. $f(x) = x^2 + 4$

## Theorem on the Exact Number of Zeros of a Polynomial

Definition: The **multiplicity of a zero** is the number of times the corresponding factor occurs in the polynomial. See the 3rd example below: the multiplicity of -1 is 2 since it's corresponding factor, ($x$+1), occurs twice in the polynomial.

**Fact:** If $f(x)$ is a polynomial function of degree $n > 0$ (and if a zero of multiplicity $m$ is counted $m$ times), then $f(x)$ has exactly $n$ zeros if we include zeros that are complex numbers.

| **Polynomial** | **Degree** | **Factored over Complex Numbers** | **Zeros** |
|---|---|---|---|
| $f(x) = x^2 + 4x - 5$ | | $f(x) = (x+5)(x-1)$ | $-5$, $1$ |
| $f(x) = x^3 - x^2 + 4x - 4$ | | $f(x) = (x+2i)(x-2i)(x-1)$ | $\pm 2i$, $1$ |
| $f(x) = x^4 + 2x^3 - 3x^2 - 8x - 4$ | | $f(x) = (x+1)^2(x-2)(x+2)$ | $-1$, $\pm 2$ |

*Find a polynomial $f(x)$ of degree 3 that has the indicated zeros and satisfies the given condition.*

2. $-2i$ , $2i$ , $0$ $f(1) = 10$

3. *Find a polynomial $f(x)$ of degree 4 such that 3 is a zero of multiplicity 2, the zeros 5 and -1 have multiplicity 1 and* $f(0) = 9$.

*Find a factored form for a polynomial function $f(x)$ graphed below that has a minimal degree. Assume that the intercept values are integers and Xscl = Yscl = 1.*

4.