Math 4 Pre-Calculus
Lecture 10 Part1 (no part 2)

Homework
pg. 386,387: 11–19 odd, 45–65 eoo

Complex and Rational Zeros of Polynomials — 5.5 and 5.6

Find all zeros.

1. $f(x) = x^3 - x^2 + 9x - 9$

Complex Conjugate Zeros

If a polynomial $f(x)$ of degree $n > 1$ has real number coefficients and if $z = a + bi$, $b \neq 0$ is a complex zero of $f(x)$, the conjugate $\overline{z} = a - bi$ is also a zero of $f(x)$.

A polynomial $f(x)$ with real coefficients and leading coefficient 1 has the given zeros and degree. Express $f(x)$ as a product of linear and quadratic polynomial factors with real coefficients that are irreducible over $\mathbb{R}$.

2. -3 , $1 - 7i$ degree 3

3. 0 , $3i$, $4 + i$ degree 5

Find all solutions of the equation.

4. $12x^3 + 8x^2 - 3x - 2 = 0$

5. $3x^5 - 10x^4 - 6x^3 + 24x^2 + 11x - 6 = 0$

Theorem on Rational Zeros of a Polynomial

If the polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ has integer coefficients and if $\frac{c}{d}$ is a rational zero of $f(x)$, such that conjugate c and d have no common prime factor, then

1. The numerator, c, of the zero is a factor of the constant term a_0.
2. The denominator, d, of the zero is a factor of the leading coefficient a_n.

In other words:

$$\text{Possible rational zeros} = \frac{\text{factors of the constant term } a_0}{\text{factors of the leading coefficient } a_n}$$

List all possible rational zeros.

6. $3x^5 - 10x^4 - 6x^3 + 24x^2 + 11x - 6 = 0$

Factors of -6:

Factors of 3:

Possible Rational zeros:

Find all solutions of the equation.

7. $3x^5 - 10x^4 - 6x^3 + 24x^2 + 11x - 6 = 0$

Use synthetic division to test a possible rational zero. Try:

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Rewrite equation:
Note: *The constant and leading coefficient are still the same as the original, so the possible rational zeros are still the same, and what didn't work previously won't work now.*

Use synthetic division to test a possible rational zero for the quotient. Try:

Rewrite equation:
Note: *The constant and leading coefficient are still the same as the original, so the possible rational zeros are still the same, and what didn't work previously won't work now.*

Use synthetic division to test a possible rational zero for the quotient. Try:

Use synthetic division to test a possible rational zero for the quotient. Try:

Rewrite equation:
Note: The quotient is now a quadratic that can be factored.

Write the equation as ***a product of linear and quadratic factors irreducible over*** $\mathbb{R}$.

Solutions *to* $3x^5 - 10x^4 - 6x^3 + 24x^2 + 11x - 6 = 0$