Math 4 Pre-Calculus
Lecture 11, Part 1 (no Part 2)

**Homework**
**pg. 352,353: 11–51eoo**
**pg. 366, 367: 7–43 eoo, 45,47**

# Rational Functions — 5.2 and 5.3

## Rational Function

A function is a rational function if $f(x) = \dfrac{g(x)}{h(x)}$, where $g(x)$ and $h(x)$ are polynomials.

**Domain**:
**Vertical asymptotes** occur at x-values for which the **denominator** $h(x)$ **equals zero,** but the numerator $g(x)$ does not equal zero for these same x-values.
**Holes** occur at the x-values for which both the **denominator** $h(x)$ **and** the **numerator** $g(x)$ **equal zero**. Find the y-value of the hole by evaluating the function at that x-value after canceling out the common factors.
**Non-vertical asymptotes** depend on the degrees of the denominator $h(x)$ and the numerator $g(x)$.
The **range** depends on the global behavior which can be determined by the asymptotes.
**y-intercept**: Plug in 0 for x to find y. In other words, find $f(0)$. The y-intercept is a point: $(0\ ,\ ?)$.
**x-intercept(s) or root(s) or zero(s)**: Find by setting $y = 0$ and solving for x. The zeros for a rational function are the zeros of the numerator (except those that are holes.) The x-intercept(s) is(are) a point(s): $(?\ ,\ 0)$.

## Vertical Asymptotes

The line $x = a$ is a vertical asymptote for the graph of a function $f$ if $f(x) \to \infty$ or $f(x) \to -\infty$ as $x$ approaches $a$ from either the left or the right. The graph will **never** cross a vertical asymptote. The *multiplicity* of the factor that causes a vertical asymptote *determines the behavior on either side of that vertical asymptote*. If the multiplicity is odd the graph behaves opposite on either side. If the multiplicity is even, the graph behaves the same on either side.

## Horizontal Asymptotes

The line $y = c$ is a horizontal asymptote for the graph of a function $f$ if $f(x)$ ® $c$, as $x \to \infty$ or as $x \to -\infty$. The graph *may* cross a horizontal asymptote, to find out if and "where", set $f(x) = c$ and solve for x.

## Oblique Asymptotes

The line $y = ax + b$ is an oblique asymptote for the graph of a function $f$ if $f(x)$ ® $ax + b$, as $x \to \infty$ or as $x \to -\infty$.

## Rational Functions—"Bottom Degree Superior"

The degree of the denominator $h(x)$ is larger than the degree of the numerator $g(x)$

**Non-vertical asymptotes (Horizontal)**: $y = 0$

In mathematical terms, this means: as $x \to \infty,\ f(x) \to 0$ and as $x \to -\infty,\ f(x) \to 0$.

## Rational Functions—"Same Degree"

The degree of the denominator $h(x)$ is the same as the degree of the numerator $g(x)$

**Non-vertical asymptotes (Horizontal)**: $y = \text{ratio of the leading coefficients}$

In mathematical terms, this means: as $x \to \infty,\ f(x) \to \text{ratio}$ and as $x \to -\infty,\ f(x) \to \text{ratio}$.

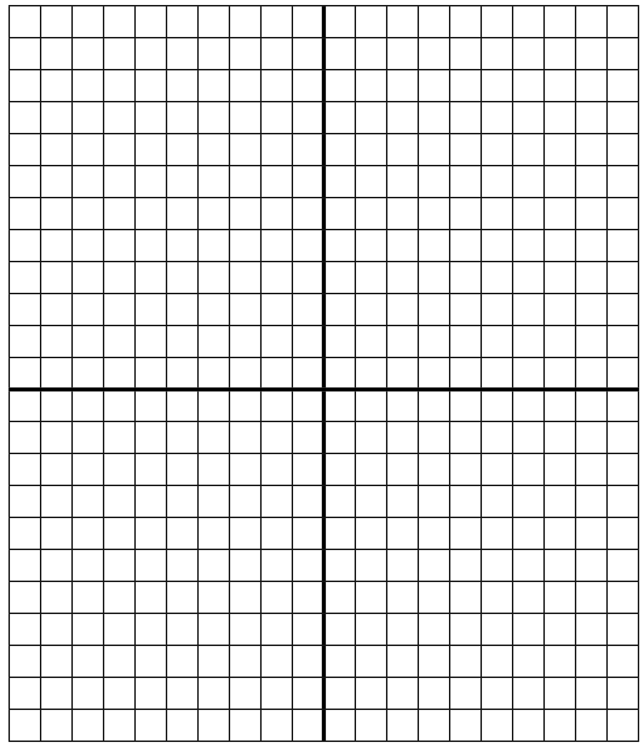
## Rational Functions—"Top Degree Superior"

The degree of the denominator $h(x)$ is smaller than the degree of the numerator $g(x)$. In this class we will only look at cases where the denominator's degree is smaller than the numerator's degree by one.

**Non-vertical asymptotes (Oblique)**: $y = ax + b$ , where $ax + b$ is the quotient of the numerator divided by the denominator.

In mathematical terms, this means: as $x \to \infty,\ f(x) \to ax + b$ and as $x \to -\infty,\ f(x) \to ax + b$.

*Graph the equation.* $f(x) = \dfrac{-2}{x-3}$



**Domain**:

**Holes**:

**Vertical Asymptotes**:

**Degree of the numerator**:

**Degree of the denominator**:

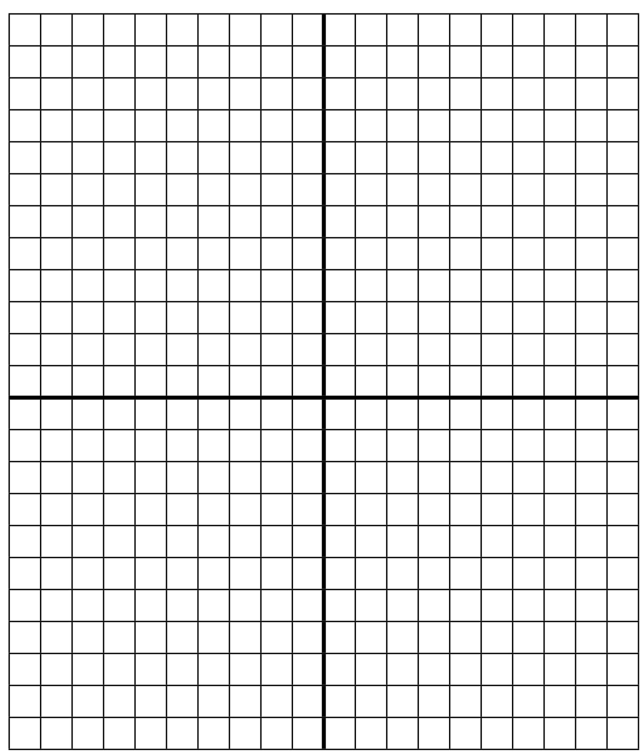
**Non-vertical Asymptotes**:

**Does graph cross the horizontal asymptote?**

**y-intercept**:

**x-intercept(s) or root(s) or zero(s)**:

*Graph the equation.* $f(x) = \dfrac{x^2 - 2x - 3}{x^3 - 9x}$



**Domain**:

**Holes**:

**Vertical Asymptotes**:

**Degree of the numerator**:

**Degree of the denominator**:

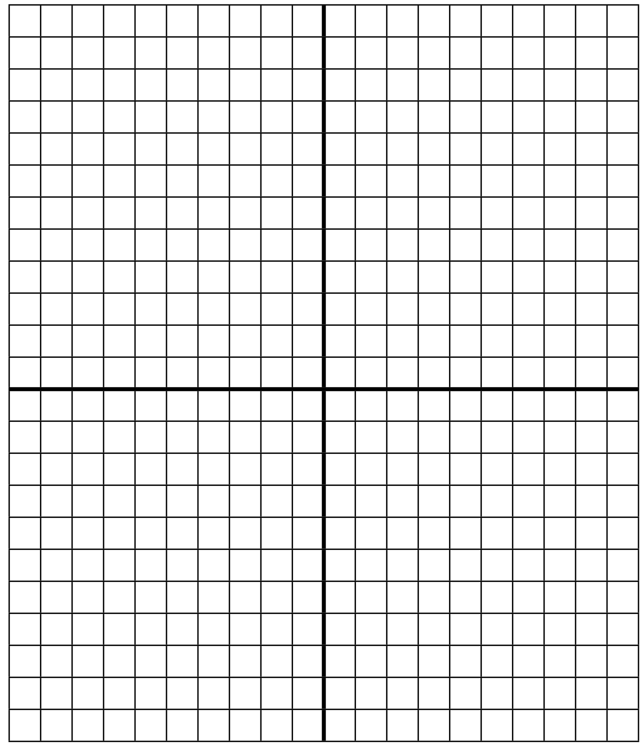
**Non-vertical Asymptotes**:

**Does graph cross the horizontal asymptote?**

**y-intercept**:

**x-intercept(s) or root(s) or zero(s)**:

*Graph the equation.* $f(x) = \dfrac{2x+5}{x-1}$



**Domain**:

**Holes**:

**Vertical Asymptotes**:

**Degree of the numerator**:

**Degree of the denominator**:

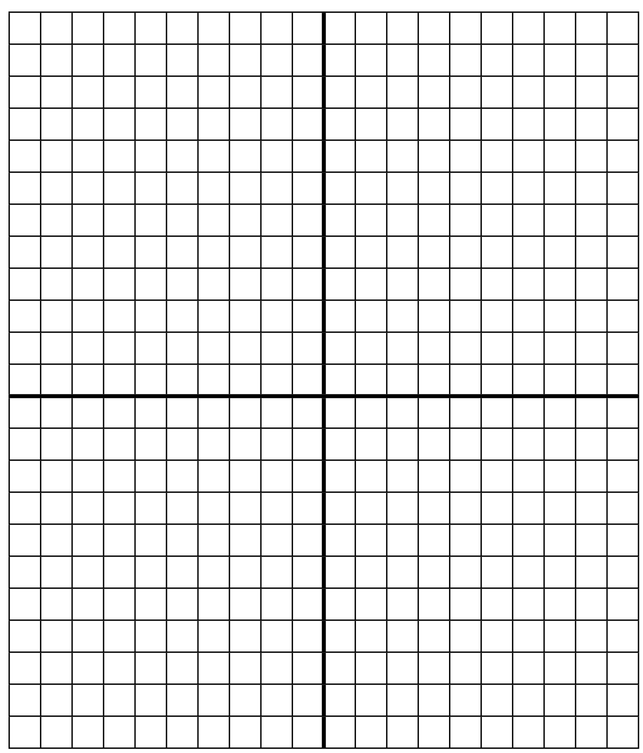
**Non-vertical Asymptotes**:

**Does graph cross the horizontal asymptote?**

**y-intercept**:

**x-intercept(s) or root(s) or zero(s)**:

*Graph the equation.* $f(x) = \dfrac{-3x^2 - 3x + 6}{(x-3)^2}$



**Domain**:

**Holes**:

**Vertical Asymptotes**:

**Degree of the numerator**:

**Degree of the denominator**:

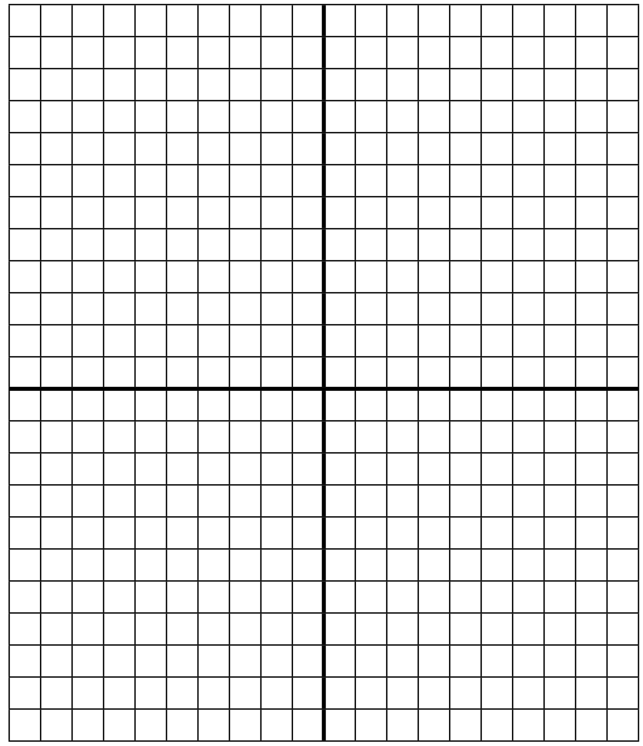
**Non-vertical Asymptotes**:

**Does graph cross the horizontal asymptote?**

**y-intercept**:

**x-intercept(s) or root(s) or zero(s)**:

*Graph the equation.* $f(x) = \dfrac{2x^2 - x - 3}{x - 2}$



**Domain**:

**Holes**:

**Vertical Asymptotes**:

**Degree of the numerator**:

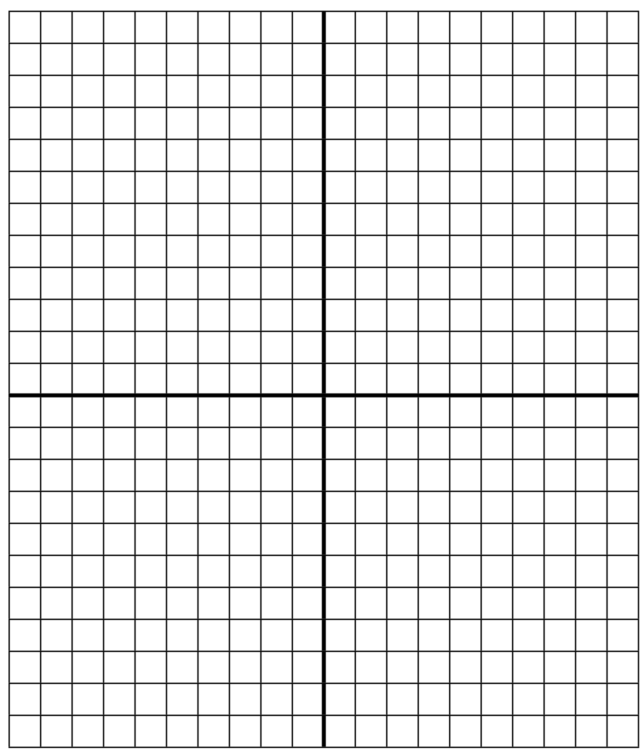
**Degree of the denominator**:

**Non-vertical Asymptotes**:

**y-intercept**:

**x-intercept(s) or root(s) or zero(s)**:

*Graph the equation.* $f(x) = \dfrac{1 - x^3}{x^2}$



**Domain**:

**Holes**:

**Vertical Asymptotes**:

**Degree of the numerator**:

**Degree of the denominator**:

**Non-vertical Asymptotes**:

**y-intercept**:

**x-intercept(s) or root(s) or zero(s)**:

*Find an equation of a rational function* $f$ *that satisfies the given conditions.*

1. vertical asymptotes: $x = -2$, $x = 4$

horizontal asymptotes: $y = 0$

x-intercept: $(\ 1\ ,\ 0\ )$ *(in the book, this will be written as : x-intercept: 1)*

$f(0) = 2$

2. vertical asymptotes: $x = 0$, $x = 1$

   horizontal asymptotes: $y = -\frac{1}{2}$

   x-intercept: $(\ -1\ ,\ 0\ )$ *(in the book, this will be written as : x-intercept: -1)*

   hole at: $x = 3$