Math 4 Pre-Calculus
Lecture 12 Part 1

Homework
pg. 433: 11,13, 29–89eoo
pg. 472: 9–25eoo
pg. 484,485: 1–11odd

Compound Interest and e — 6.3, 6.7

Use the simple interest formula to fill in the following table, if $P is invested, and interest is compound 1 time per year at an annual interest rate r.

t	A (amount in account after t years)
0	P
½	
1	$P + I$ or $P + \mathrm{Pr}$ or $P(1+r)$
2	$P(1+r) + P(1+r)r$ or $P(1+r)^2$
3	$P(1+r)^3$
t	

Compound Interest Formula

$$A = P\left(1+\frac{r}{n}\right)^{nt},$$

where
$P =$ principal
$r =$ annual interest rate expressed as a decimal
$n =$ number of interest periods per year
$t =$ number of years P is invested
$A =$ amount after t years

1. **Compound Interest**
   If $5,000 is invested at a rate of 6% per year compounded quarterly, find the principal (amount in the account) after

   a) 3 months

   b) 3 years

Use the compound interest formula to fill in the following table, if $1 is invested at an annual interest rate of 100% for 1 year for the various numbers of compounding periods per year.

n	$A = 1\left(1+\frac{1}{n}\right)^{n\times 1}$ or $A = \left(1+\frac{1}{n}\right)^{n}$
1	$(1+1)^1 =$
4	$\left(1+\frac{1}{4}\right)^4 =$
12	$\left(1+\frac{1}{12}\right)^{12} =$
56	$\left(1+\frac{1}{56}\right)^{56} =$
365	$\left(1+\frac{1}{365}\right)^{365} =$
1,000	$\left(1+\frac{1}{1,000}\right)^{1,000} =$
10,000	$\left(1+\frac{1}{10,000}\right)^{10,000} =$
100,000	$\left(1+\frac{1}{100,000}\right)^{100,000} =$
1,000,000	$\left(1+\frac{1}{1,000,000}\right)^{1,000,000} =$

What can you conclude about the value of the expression $\left(1+\frac{1}{n}\right)^n$ as n gets really large?

In mathematical terms, as $n \to \infty$, $\left(1+\frac{1}{n}\right)^n$ ®

The Number e

If n is a positive integer, then as $n \to \infty$, $\left(1+\frac{1}{n}\right)^n \to e \approx 2.71828$

Show what happens to the compounded interest formula $A = P\left(1+\frac{r}{n}\right)^{nt}$, as $n \to \infty$?

Let $k = \frac{n}{r}$ and note that as $k \to \infty$, $n \to \infty$.

Continuously Compounded Interest Formula

$A = Pe^{rt}$, where
P = principal
r = annual interest rate expressed as a decimal
t = number of years P is invested
A = amount after t years

2. **Continuously Compounded Interest**
How much money, invested at an interest rate of 4% per year compounded continuously, will amount to \$25,000 in 3 years?

Law of Exponential Growth (or Decay) Formula

Let q_0 be the value of a quantity q at time (that is, q_0 is the initial amount of q). If q changes instantaneously at a rate proportional to its current value, then

$$q = q(t) = q_0 e^{rt} \quad ,$$

where $r > 0$ is a rate of growth (or $r < 0$ is the rate of decay) of q.

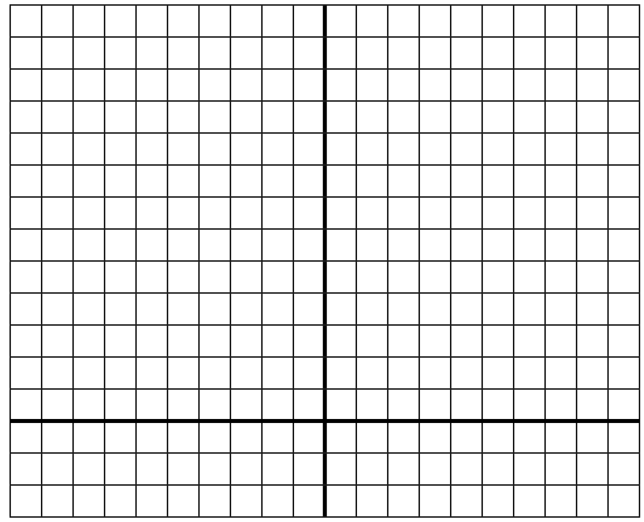
Note: The above formula is the "same" as the continuously compounded interest formula.

3. **Population Growth Rate**
The 1985 population estimate for India was 762 million, and the population has been growing continuously at a rate of about 2.2% per year. Assume that this rapid growth rate continues.

a) Write an exponential function to model India's population as a function of years since 1985.

b) Use the model above to predict India's population in 2010.

Natural Exponential Function $f(x) = e^x$ (Note: $e \approx 2.71828$)



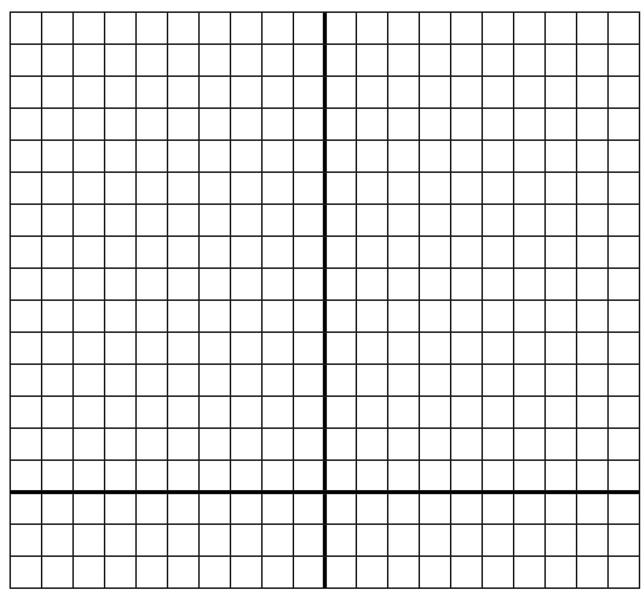
Domain:

Horizontal Asymptote at $y = 0$.

Range:

The natural exponential function contains the points $(0\ ,\ 1)$ and $(1\ ,\ e)$.

4. *Graph the equation.*
   $f(x) = e^{-x} + 2$



Domain:

Transformations:

Points:

Range:

Solve the equation.

5. $e^{-x}e^{2} = \left(\frac{1}{e}\right)^{4x+12}$

Find all zeros of f .

6. $f(x) = x^2\left(2e^{2x}\right) + 2xe^{2x} + e^{2x} + 2xe^{2x}$