Math 4 Pre-Calculus
Lecture 12 Part 2

Homework
See homework assigned for Lecture 12 Part 1
pg. 435: 95–109 odd

Exponential Functions — 6.3

Exponential Function

$f(x) = a^x$, $a > 0$ and $a \neq 1$

Domain:
Horizontal Asymptote at $y = 0$. The horizontal asymptote will move up or down depending on a horizontal shift.

The **range** depends on the global behavior which can be determined by the horizontal asymptote and the sign of vertical stretch/compression factor, denoted by the letter *b* in your book..
All "basic" exponential functions contain the points $(0\ ,\ 1)$ and $(1\ ,\ a)$.

Exponential Growth Functions

$f(x) = a^x$, $a > 1$

Domain:

Graph:

If $a > 1$, the graph is a " shape : right side up" . **Range**:
In mathematical terms, this means: as $x \to \infty,\ f(x) \to \infty$ and as $x \to -\infty,\ f(x) \to 0^+$

Exponential Decay Functions

$f(x) = a^x$, $0 < a < 1$

Domain:

Graph:

If $0 < a < 1$, the graph is a " shape : left side up" . **Range**:
In mathematical terms, this means: as $x \to \infty,\ f(x) \to 0^+$ and as $x \to -\infty,\ f(x) \to \infty$

1. *Graph the equation.*
   $f(x) = 3^x$

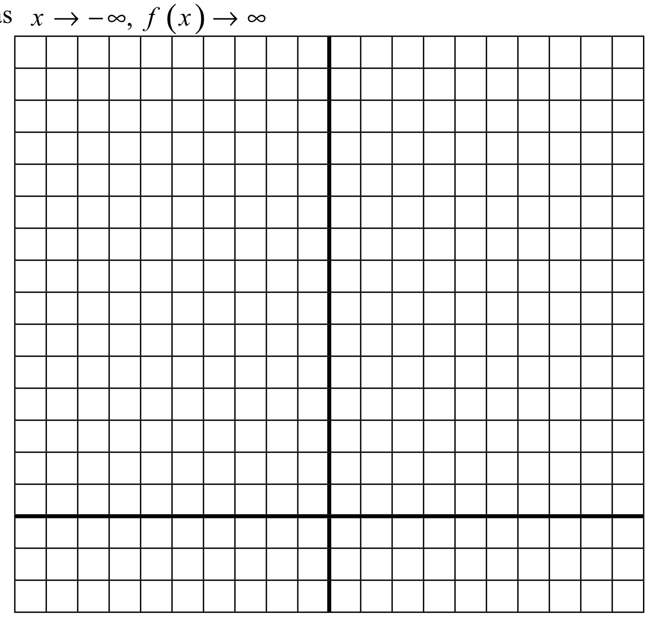
   Domain:

   Value of base:

   Horizontal Asymptote:

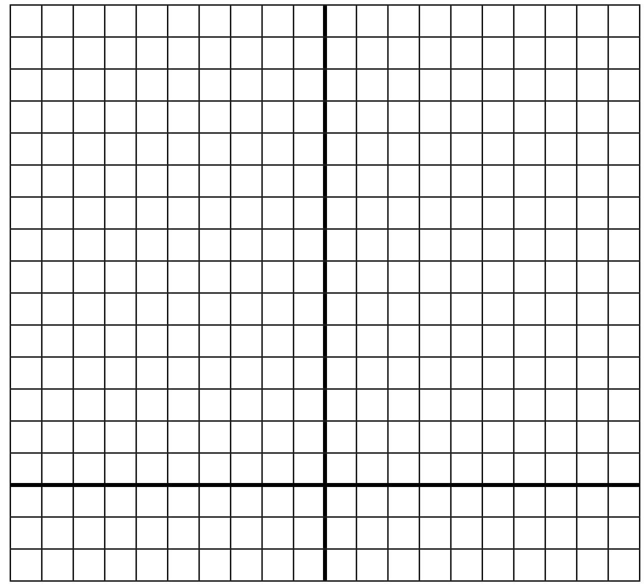
   Range:

   Points:



2. *Graph the equation.*

$f(x) = -3^{|x|} - 2$



Domain:

Value of base:

Horizontal Asymptote:

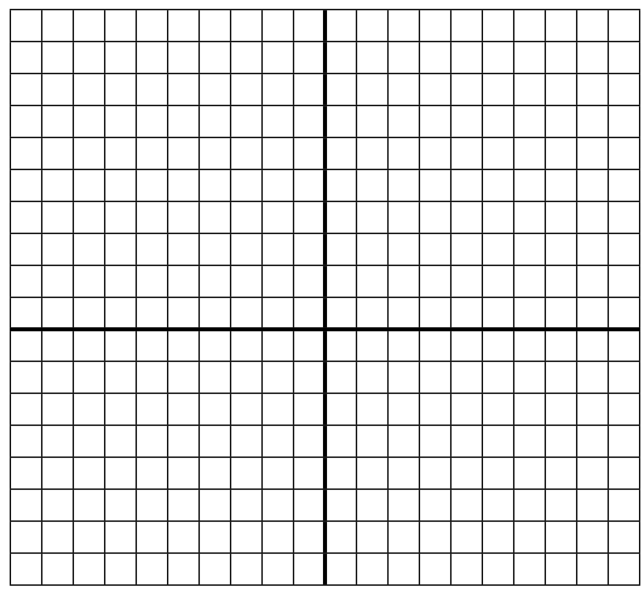
Transformations:

Range:

Points:

3. *Graph the equation.*

$f(x) = \left(\frac{1}{2}\right)^{x-1}$



Domain:

Value of base:

Horizontal Asymptote:

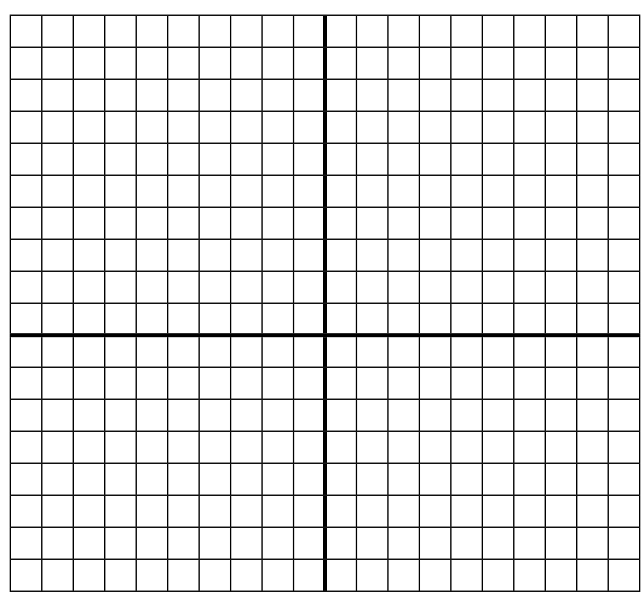
Transformations:

Range:

Points:

4. *Graph the equation.*

$f(x) = \left(\frac{1}{2}\right)^{-x} + 3$



Domain:

Value of base:

Horizontal Asymptote:

Transformations:

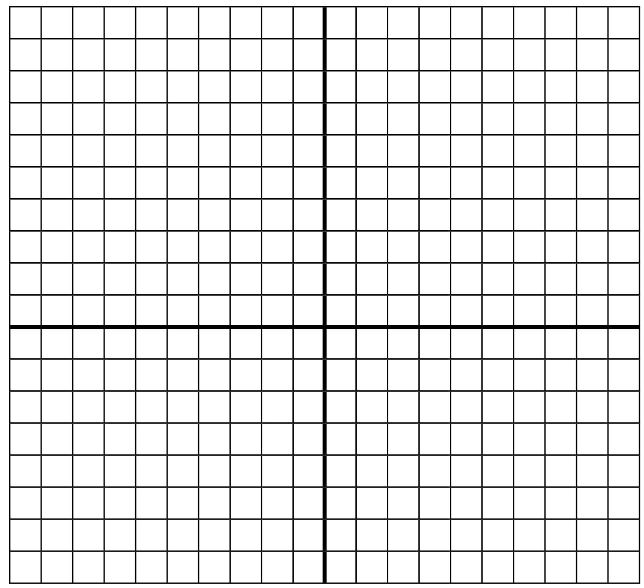
Range:

Points:

Find an exponential function of the form $f(x) = ba^x$ or $f(x) = ba^x + c$ *that has the given graph or conditions.*

5. y-intercept $(0, 4)$; passes through point $P(2, 16)$ and has a horizontal asymptote at $y = -1$

6. Graph the function



Exponential Functions Are One-to-One

The exponential function f given by $f(x) = a^x$ for $0 < a < 1$ or $a > 1$ is one-to-one. Thus, the following equivalent conditions are satisfied for real numbers x_1 and x_2

1. If $x_1 \neq x_2$, then $a^{x_1} \neq a^{x_2}$.
2. If $a^{x_1} = a^{x_2}$, then $x_1 = x_2$

Solve each equation.

7. $4^{x^2} = \left(\frac{1}{2}\right)^{3x}$

8. $25^x \cdot \left(\frac{1}{5}\right)^{4-2x} = \left(5^x\right)^3 \cdot 125$

Exponential Decay Applications

The **half-life** of a substance (or population) with an exponential decay is a defining characteristic. The half-life of a substance (or population) is the time it takes for one-half of the original amount in a given sample to decay. The half-life of an isotope distinguishes one isotope from another and is often used in carbon-dating of an object.

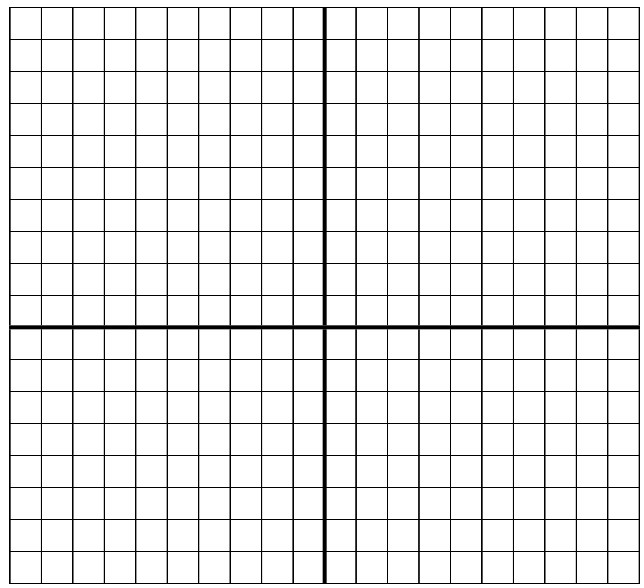
9. **Exponential Decay**

   Let Q represent a mass of carbon 14 (in grams), whose half-life is 5730 years. The quantity of carbon 14 present after t years is given by $Q(t) = 10\left(\frac{1}{2}\right)^{t/5730}$.

   a) Determine the initial quantity of carbon 14 (when $t = 0$).

   b) Predict the amount of carbon 14 present after 2000 years.

   c) Sketch a graph of $Q(t)$ at any time from $t = 0$ to $t = 10,000$.



Exponential Growth Applications

The **doubling-time** of a substance (or population) with an exponential growth is a defining characteristic. The doubling-time of a substance (or population) is the time it takes for the substance (or population) to double its original amount.

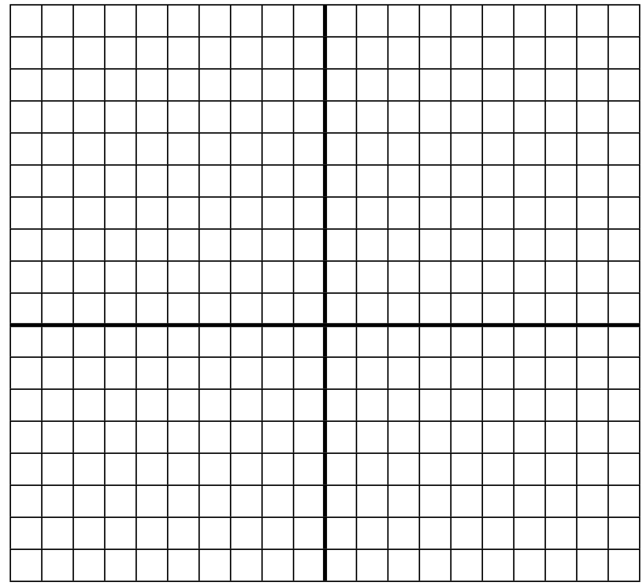
10. **Exponential Growth**

    A certain type of bacterium increases according to the model $P(t) = 100\left(\frac{5}{4}\right)^{t}$, where t is the time in hours.

    a) Determine the initial amount of bacterium.

    b) Predict the amount of bacterium after 5 hours.

c) Sketch a graph of $P(t)$ at any time from $t = 0$ to $t = 10$.



d) Use the above graph to estimate the doubling-time of $P(t)$.

11. **Exponential Modeling**
In a research experiment, a population of fruit flies is increasing at an exponential growth rate. Initially there are 30 fruit flies. After 2 days, there are 100 flies.

a) Find a simple exponential function $y = b a^x$ that models the population of the fruit flies after x days.

b) Use the above model to predict the number of fruit flies after 10 days.