Homework
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Math 4 Pre-Calculus
Lecture 14 Part 1

Change of Base &
Exponential and Logarithmic Equations— 6.6

Solve. $5^x = 7$

Change of Base Formula

If $u > 0$ and if a and b are positive real numbers different from 1, then:

$$\log_b u = \frac{\log_a(u)}{\log_a(b)} \qquad \log_b u = \frac{\log(u)}{\log(b)} \qquad \log_b u = \frac{\ln(u)}{\ln(b)}$$

Approximate each logarithm. Give your answer accurate to 3 decimal places.

1. $\log_2 3$

2. $\log_{\frac{1}{2}} 9$

Some types of exponential equations:

1. *Two expressions, which only consist of factors, are equal to each other. Rewrite all factors so that the bases are the same, use rules of exponents to get two expressions with the same base equal each other, and solve by setting the exponents equal to each other.*

   Example: $25^x \times \left(\frac{1}{5}\right)^{4-2x} = \left(5^x\right)^3 \times 125$

2. *One expression with a variable in the exponent is equal to a constant. Rewrite in logarithmic form.*

   Example: $e^{x^2} = 16$

3. *Two expressions, which have unlike bases, are equal to each other. Take the common logarithm (or natural logarithm) of both sides, use the properties of logs to bring down the exponent, and solve the remaining equation.*

   Example: $5^{2x+1} = 3^{x-2}$

4. *An equation may be a "quadratic in disguise". Add or subtract to get a zero on one side. Use substitution to solve the "quadratic", then solve for the original equation's variable.*

   Example: $2^x - 6\left(2^{-x}\right) = -5$

5. *An equation may be one that has two or more exponential terms, which have the same base, but is not a "quadratic in disguise". Add or subtract to get a zero on one side. Factor. Solve by setting each factor equal to zero.*

   Example: $-2xe^{2x} = x^2\left(2e^{2x}\right) + 2xe^{2x} + e^{2x}$

Some types of logarithmic equations:

1. *Two logarithms, which have the same base and no roots or powers of logarithms, are equal to each other. Solve by setting the arguments of the logarithms equal to each other.*

   Example: $\log_6(4x-5) = \log_6(2x+1)$

2. *One logarithmic expression with a variable in the argument is equal to a non-logarithmic constant. Rewrite in exponential form. Solve the resulting equation.*

   Example: $\log(x+2) = 4$

3. *An equation consists of two or more logarithms with the same base, and one or more non-logarithmic constants. Add and subtract terms to get the logarithms on one side of the equation and the non-logarithmic constants on the other side. Use the properties of logarithms to write the logarithms as a single logarithm. Rewrite in exponential form. Solve the resulting equation.*

   Example: $\log(x+3) - 2 = \log(x)$

4. *Two logarithms, which have the same base and some kind of root or powers of logarithms, are equal to each other. Use the properties of logarithms to remove the powers from the argument. If there are any roots of logarithms raise each side to an appropriate power to undo. Add or subtract to get a zero on one side. Factor. Solve by setting each factor equal to zero.*

   Example: $\ln\sqrt[4]{x} = \sqrt{\ln x}$

Find the exact solutions and then give approximate solutions accurate to two decimal places.

3. $5^{2x+1} = 3^{x-2}$

4. $\log(x-15) - 2 = -\log(x)$

5. $3 \times 3^{x} + 9 \times 3^{-x} = 28$

6. $\ln \sqrt[4]{x} = \sqrt{\ln x}$

7. **Compound Interest**

Use the compound interest formula to determine how long it will take for a sum of money to double if it is invested at a rate of 4% per year compounded monthly.

8. **Depreciation**

The value V of a Chevy Cavalier Coupe that is *t* years old can be modeled by $V(t) = 14{,}512(0.82)^t$

a) Determine the value of the Chevy when it was purchased.

b) Determine the value of the Chevy after 2 years.

c) Determine when the value of the Chevy is $1,000.

d) What is the "half-life" of the Chevy?