Homework
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Angles — 7.1

Definitions

In *geometry*, an **angle** is the set of points determined by two rays or half-lines. See picture below of angle AOB (denoted $\angle AOB$)
In *trigonometry*, an **angle** is a rotation of a ray and has a direction.

Coterminal angles have the same initial side and same terminal side. See picture below.

A **straight angle** is an angle whose sides lie on the same straight line but extend in opposite directions from its vertex.

An angle in **standard position** on a coordinate system is an angle whose vertex is at the origin and whose initial side is the positive x-axis.. A **positive angle** is formed by a terminal side rotating counterclockwise. A **negative angle** is formed by a terminal side rotating clockwise.

Lowercase Greek letters such as α (alpha), β (beta), γ (gamma), θ (theta), ϕ (phi), and so on are often used to denote angles.

One unit of measurement for angles is the **degree**, denoted by °. There are 360° in one complete counterclockwise revolution; therefore, one degree is $\frac{1}{360}$ of one complete counterclockwise revolution. A degree can be divided into smaller units of measure. One degree is 60 **minutes**, denoted by $60'$. One minute is 60 **seconds**, denoted by $60''$. A measurement of $59°10'45''$ is read "59 degrees, 10 minutes, 45 seconds". Degree measure is often used in surveying, navigation and mechanical design.

A **right** angle is half of a straight angle and has measure 90°.

An **acute** angle is an angle with a measure between 0° and 90°. For example, $\alpha = 35°$ is an acute angle.

An **obtuse** angle is an angle with a measure between 90° and 180°. For example, $\alpha = 120°$ is an obtuse angle.

The sum of the measures of two **complementary** angles is 90°. For example, $\alpha = 34°50'$ and $\beta = 55°10'$ are complementary angles.

The sum of the measures of two **supplementary** angles is 180°. For example, $\alpha = 34°50'$ and $\beta = 145°10'$ are supplementary angles.

1. Find two **positive co-terminal** angles of 115°. Illustrate this concept with a picture.

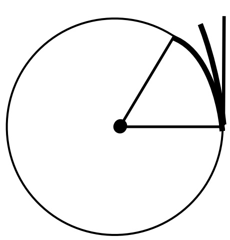
2. Find two **negative co-terminal** angles of 25°. Illustrate this concept with a picture.

3. Find the angle that is **complementary** to 34.2°. Illustrate this concept with a picture.

4. Find the angle that is **supplementary** to $59^\circ 10' 45''$. Illustrate this concept with a picture.

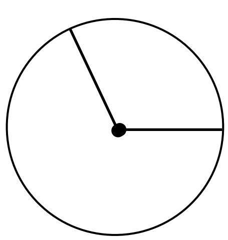
5. Express $59^\circ 10' 45''$ as a decimal to the nearest ten-thousandth of a degree.

Another unit of measurement for angles is the **radian**, which is based on a circle's radius. A central angle of a circle is an angle whose vertex is at the center of the circle. One **radian** is the measure of the central angle of a circle subtended by an arc equal in length to the radius of the circle. See picture below.

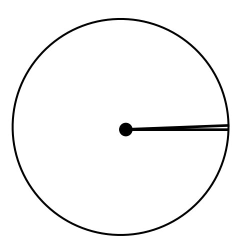


Take one line segment with the same length as the radius, put one end on the circle and then bend it onto the circle. The angle it makes is **one radian**

When radian measure is used, no units are added. So an angle of measure 2 is an angle with measure 2 radians, which is very different from an angle of measure 2°. See pictures below.



2 radians: 2
(no units)



2 degrees: 2°

There are 2π radians (approximately 6.28 radians) in a complete revolution.

Relationship between Degrees and Radians

$$\pi \text{ radians} = 180°$$

To convert from degrees to radians, multiply by $\frac{\pi}{180°}$.

To convert from radian to degrees, multiply by $\frac{180°}{\pi}$.

6. Find the exact radian measure of 405°. Draw this angle on a coordinate plane.

7. Find the exact degree measure of the angle $-\frac{2\pi}{3}$. Draw this angle on a coordinate plane.

8. Write $\theta = 4$ in terms of degrees, minutes and seconds.

The circumference of a circle of radius r is the distance around the circle. This circumference can be thought of as a circular arc that forms a complete circle. It can be found by the formula: $C = 2\pi r$. If the circle is formed by rotating a central angle θ counterclockwise for one complete revolution, then $\theta = 2\pi$, and the formula would turn into $C = \theta r$. The formula below shows that this follows for any central angle of a circle and its corresponding arc.

Formula for the Length of a Circular Arc

If an arc of length s on a circle of radius r subtends a central angle of **radian** measure θ, then

$$s = r\theta$$

The area of a circle of radius r can be thought of as the area created by a circular arc that forms a complete circle. It can be found by the formula: $A = \pi r^2$. If the circle is formed by rotating a central angle θ counterclockwise for one complete revolution, then $\theta = 2\pi$, and the formula would turn into $A = \frac{1}{2}\theta r^2$. The formula below shows that this follows for any central angle of a circle and its corresponding arc.

Formula for the Area of a Circular Sector

If θ is the **radian** measure of a central angle of a circle of radius r and if is the area of the circular sector determined by A, then

$$A = \frac{1}{2}r^2\theta$$

9. Find both a) the length of the arc of the colored sector in the figure, and b) the area of the colored sector.

10. a) Find the radian and degree measure of the central angle θ subtended by the given arc of length s on a circle of radius r. b) Find the area of the sector.

$s = 3 \text{ ft}, \qquad r = 20 \text{ in}$