Homework
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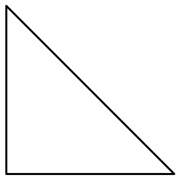
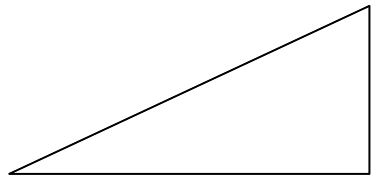
Math 4 Pre-Calculus
Lecture 15 Part 2

Trigonometric Functions of Right Triangles — 7.2

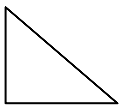
Definitions

A triangle is a **right** triangle if one of its angles is a right angle; that is, 90° . See pictures below.

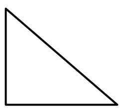




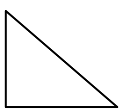
The **sine** of θ , denoted $\sin\theta$, is the ratio of the side opposite θ to the hypotenuse.



The **cosine** of θ , denoted $\cos\theta$, is the ratio of the side adjacent θ to the hypotenuse.



The **tangent** of θ , denoted $\tan\theta$, is the ratio of the side opposite θ to the side adjacent θ .

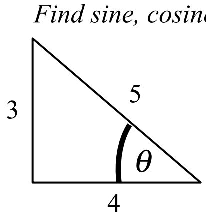


In summary:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

You can remember this by: SOH CAH TOA

1. *Find sine, cosine and tangent for the angle* θ .



2. *If* θ *is an acute angle and* $\sin\theta = \frac{3}{4}$ *, find* $\cos\theta$ *and* $\tan\theta$.

Reciprocal Trigonometric Functions

The **cosecant** of θ, denoted $\csc\theta$, is the reciprocal of the sine θ or the ratio of the hypotenuse to the side opposite θ.

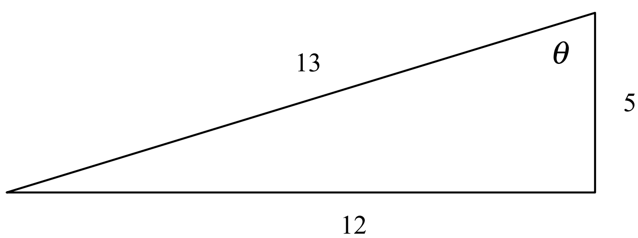
The **secant** of θ, denoted $\sec\theta$, is the reciprocal of the cosine θ or the ratio of the hypotenuse to the side adjacent θ.

The **cotangent** of θ, denoted $\cot\theta$, is the reciprocal of the tangent θ or the ratio of the side adjacent θ to the side opposite θ.

In summary:

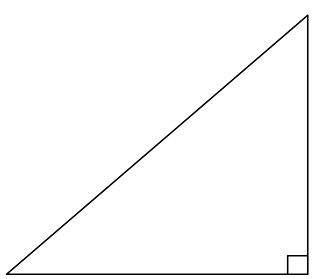
$$\csc\theta = \frac{1}{\sin\theta} = \frac{hyp}{opp} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{hyp}{adj} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{adj}{opp}$$

3. *Find all six trigonometric functions for the angle* θ.

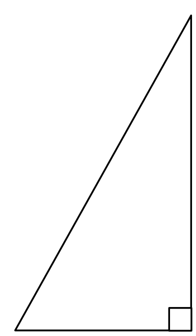


NOTE: Examples 1 and 3 are called **Pythagorean Triples** because the sides are whole numbers that satisfy the Pythagorean Theorem:
$$(one\ side)^2 + (other\ side)^2 = (hypotenuse)^2$$
There is a way to generate all of these triples:
For any 2 numbers, *m* and *n*
the two sides are: $n^2 - m^2$ and $2mn$
and the hypotenuse is: $n^2 + m^2$
See the end of this Lecture for some examples...

4. *Label all of the angles of the two triangles both in degrees and radians, and then label the lengths of each side.*



Isosceles Right Triangle

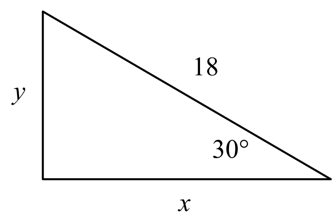


30-60-90 Triangle

Use the isosceles right triangle and the 30-60-90 triangle above to fill in the chart below.

θ *(radians)*	θ *(degrees)*	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
0							
	30°						
$\frac{\pi}{4}$							
	60°						

5. *Find exact values of* x *and* y.



6. **Distance to Mt. Fuji** The peak of Mt. Fuji in Japan is approximately 12,400 feet high. A trigonometry student, several miles away notes that the angle between level ground and the peak is 30°. Estimate the distance from the student to the point on level ground directly beneath the peak.

7. *Approximate to four decimal places, when appropriate.*

a) $\sin 75°$

b) $\cot\frac{\pi}{5}$

Fundamental Trigonometric Identities

Reciprocal Identities:

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Tangent and Cotangent Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities:

$\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

8. *Show* $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

9. *Show* $\sin^2\theta + \cos^2\theta = 1$

10. *Use the Pythagorean identities to write the expression as an integer.*

$$3\csc^2\left(\beta/2\right) - 3\cot^2\left(\beta/2\right)$$

11. *Simplify the expression.*

$$\frac{\csc\theta + 1}{\left(1/\sin^2\theta\right) + \csc\theta}$$

12. *Use the fundamental identities to write the first expression in terms of the second, for any acute angle* θ *.*

$\cos\theta, \ \sin\theta$

13. *Verify the identity by transforming the left-hand side into the right-hand side.*

$\tan\theta \cot\theta = 1$

14. *Verify the identity by transforming the left-hand side into the right-hand side.*

$\cos^2(2\theta) - \sin^2(2\theta) = 2\cos^2(2\theta) - 1$

15. *Verify the identity by transforming the left-hand side into the right-hand side.*

$$\cot\theta + \tan\theta = \csc\theta \sec\theta$$

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Some Pythagorean Triples:

	m= 1	2	3	4	5	6	7	8	9
n=									
2	[3,4,5]								
3	[8,6,10]	[5,12,13]							
4	[15,8,17]	[12,16,20]	[7,24,25]						
5	[24,10,26]	[21,20,29]	[16,30,34]	[9,40,41]					
6	[35,12,37]	[32,24,40]	[27,36,45]	[20,48,52]	[11,60,61]				
7	[48,14,50]	[45,28,53]	[40,42,58]	[33,56,65]	[24,70,74]	[13,84,85]			
8	[63,16,65]	[60,32,68]	[55,48,73]	[48,64,80]	[39,80,89]	[28,96,100]	[15,112,113]		
9	[80,18,82]	[77,36,85]	[72,54,90]	[65,72,97]	[56,90,106]	[45,108,117]	[32,126,130]	[17,144,145]	
10	[99,20,101]	[96,40,104]	[91,60,109]	[84,80,116]	[75,100,125]	[64,120,136]	[51,140,149]	[36,160,164]	[19,180,181]