Math 4 Pre-Calculus
Lecture 17

Homework
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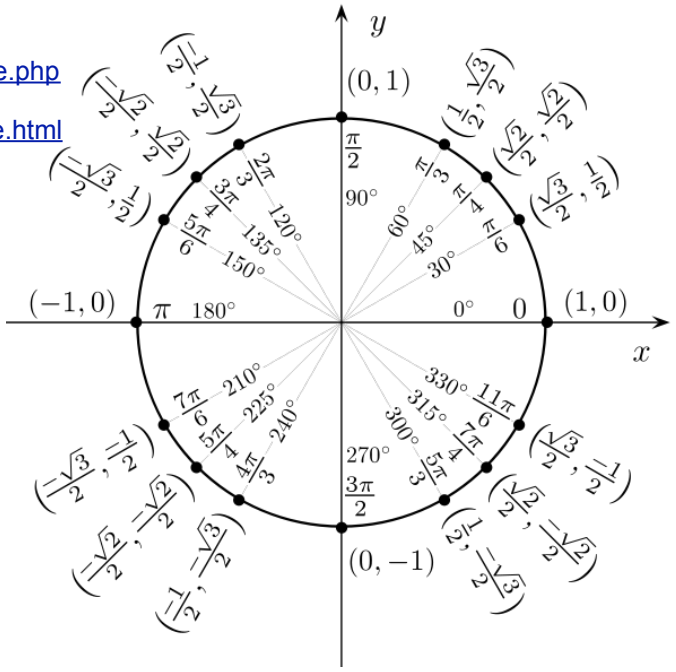
Trigonometric Functions — 7.5

The Unit Circle

The unit circle is a circle centered at the origin with a radius of one. On the unit circle, the length of an arc, s, subtended by an angle of t radians is equal to t.

http://www.dudefree.com/Student_Tools/materials/precalc/unit-circle.php
http://www.mathsisfun.com/geometry/unit-circle.html
http://www.mathlearning.net/learningtools/Flash/unitCircle/unitCircle.html

Unit Circle Exploration:
http://www.keymath.com/x3359.xml
http://www.yenka.com/freecontent/item.action?quick=jc



Definition of the Trigonometric Functions in Terms of the Unit Circle

If t is real number and $P(x, y)$ is point on the unit circle U that corresponds to t, then

$\sin t = y$ $\qquad \cos t = x$ $\qquad \tan t = \frac{y}{x}, \quad \text{if } x \neq 0$

$\csc t = \frac{1}{y}, \quad \text{if } y \neq 0$ $\qquad \sec t = \frac{1}{x}, \quad \text{if } x \neq 0$ $\qquad \cot t = \frac{x}{y}, \quad \text{if } y \neq 0$

Animated Sine and Cosine functions:
http://www.keymath.com/x3359.xml
http://www.yenka.com/freecontent/item.action?quick=je#

1. *A point $P(x, y)$ is shown on the unit circle U corresponding to a real number t. Find the values of the trigonometric functions at t.*

2. *Let* $P(t)$ *be the point on the unit circle* U *that corresponds to* t. *If* $P(t)$ *has the given rectangular coordinates, find* *a)* $P(t+\pi)$ *b)* $P(t-\pi)$ *c)* $P(-t)$ *and* *d)* $P(-t-\pi)$

$$\left(-\frac{8}{17}, \frac{15}{17}\right)$$

Periodic Functions

A function f is periodic if there exists a positive real number k such that

$$f(t+k) = f(t)$$

for every t in the domain of f. The least such positive real number k, if it exists, is the period of f.

Periodic Trigonometric Functions

If n is any integer, then

$\sin(t+2\pi n) = \sin t$	The period of a sine function is 2π.
$\cos(t+2\pi n) = \cos t$	The period of a cosine function is 2π.
$\tan(t+\pi n) = \tan t$	The period of a tangent function is π.

See the periodic behavior of sine and cosine functions at:
http://www.keymath.com/x3359.xml

Sine and Cosine Values

Note that for right triangles, sine is the ratio between the opposite side of an acute angel and the hypotenuse. This means that the absolute value of the denominator is always larger than or equal to the absolute value of the numerator; therefore the overall absolute value of the ratio is between zero and one. The same is true for the absolute value of the cosine ratio. The unit circle also supports the following conclusion:

$$-1 \le \sin t \le 1 \qquad -1 \le \cos t \le 1$$

The sine and cosine functions can be graphed individually on coordinate systems where the *x*-axis corresponds to the angle *x* and the *y*-axis corresponds to $\sin x$ or $\cos x$.

3. *Graph the equation.*

$f(x) = \sin x$

Sine Wave

Domain:

Range:

Period:

y-intercept:

x-intercepts:

Symmetry:

4. *Graph the equation.*

$f(x) = \cos x$

Cosine Wave

Domain:

Range:

Period:

y-intercept:

x-intercepts:

Symmetry:

5. *Refer to the graph of* $y = \cos x$ *to find the exact values of x in the interval* $[0\ ,\ 4\pi]$ *that satisfy the equation.*

$\cos x = -\frac{1}{2}$

Formulas for Negatives

$\sin(-t) = -\sin t$ $\quad$ $\cos(-t) = \cos t$ $\quad$ $\tan(-t) = -\tan t$

$\csc(-t) = -\csc t$ $\quad$ $\sec(-t) = \sec t$ $\quad$ $\cot(-t) = -\cot t$

6. *Use the formula for negatives to find the exact value.*

a) $\sin\left(-\frac{\pi}{6}\right)$

b) $\sec\left(-\frac{\pi}{4}\right)$

7. *Verify the identity by transforming the left-hand side into the right-hand side.*

$$\frac{\sec(-x)}{\tan(-x)} = -\csc x$$

Below is a link to animations of the tan, cot, sec, and csc functions (the animations are a little ways down the page)
http://www.intmath.com/Trigonometric-graphs/4_Graphs-tangent-cotangent-secant-cosecant.php

8. *Graph the equation.*

$f(x) = \tan x$

Tangent Function

Domain:

Vertical Asymptotes:

Range:
Period:

y-intercept:

x-intercepts:

Symmetry:

9. *Graph the equation.*
$f(x) = \cot x$

Cotangent Function

Domain:

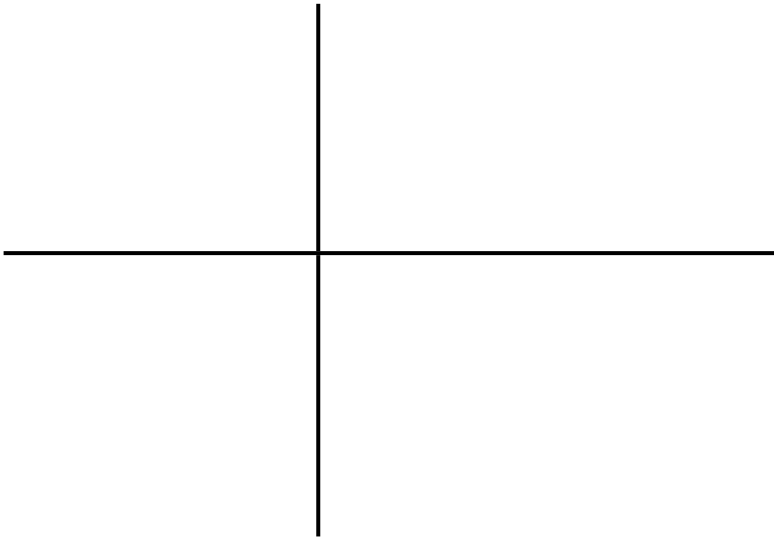
Vertical Asymptotes:

Range:
Period:

y-intercept:

x-intercepts:

Symmetry:



10. *Graph the equation.*
$f(x) = \csc x$

Cosecant Function

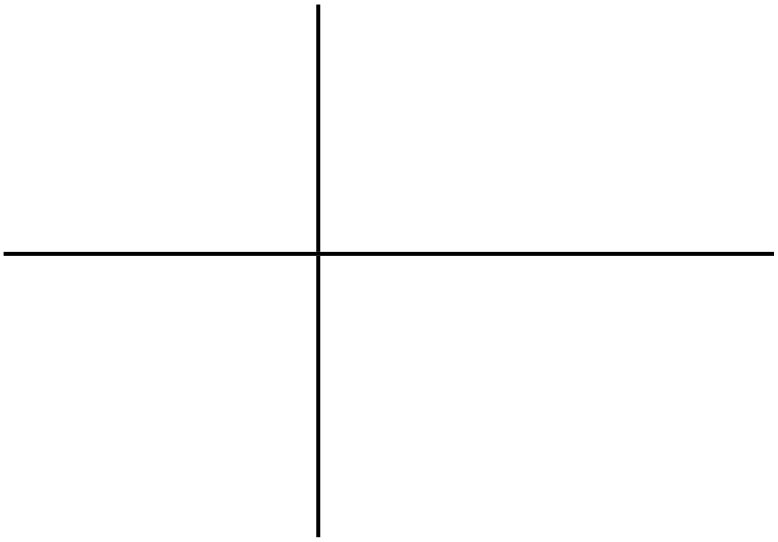
Domain:
Vertical Asymptotes:

Range:

Period:

y-intercept:

Symmetry:



11. *Graph the equation.*

$f(x) = \sec x$ **Secant Function**

Domain:

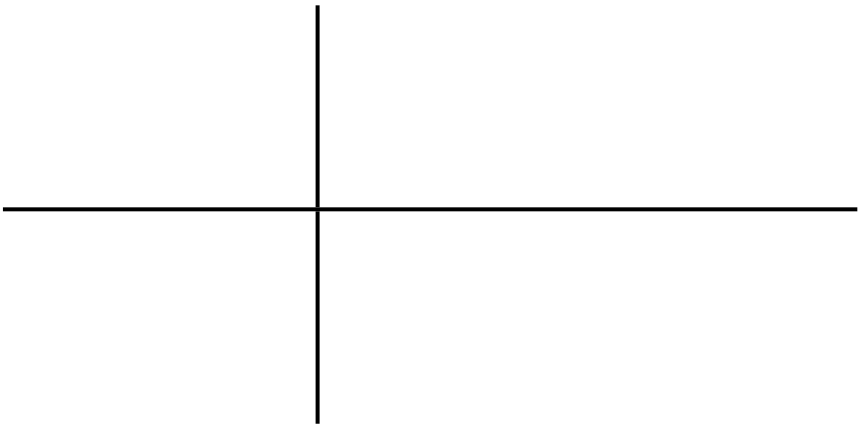
Vertical Asymptotes:

Range:

Period:

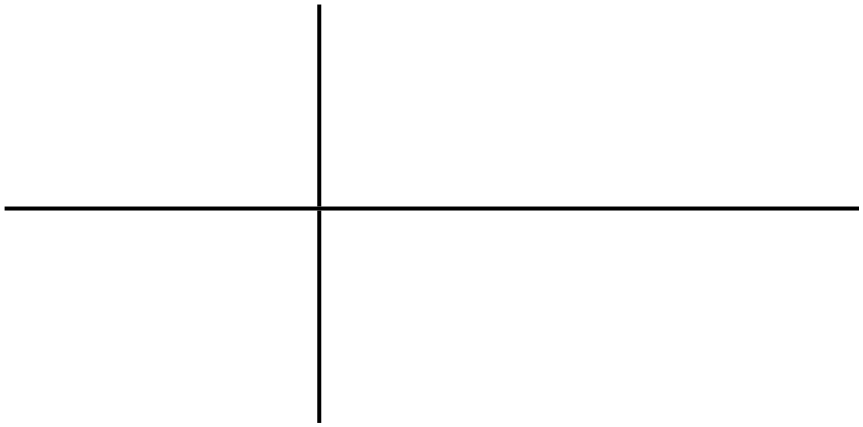
y-intercept:

Symmetry:



12. *Refer to the graph of the equation on the specified interval. Find all values of x such that for the real number a,*

a) $y = a$ *b)* $y > a$ *and*



c) $y < a$.

$y = \sin x$; $[0, 4\pi]$; $a = -\frac{\sqrt{2}}{2}$