

## Transformations of Sine and Cosine — 7.6 and 7.8

### Vertical Transformations

To compute both a vertical stretch/flip/shrink and a vertical shift on the same graph, FIRST stretch/flip/shrink, then shift.

### Theorem on Amplitudes and Vertical Shifts

If  $y = a \sin(x) + d$  or  $y = a \cos(x) + d$  for any real numbers  $a$  and  $d$ ,  $a \neq 0$ , then the graph has an amplitude of  $|a|$  and an “equilibrium” at the horizontal line  $y = d$ .

**Amplitude animation:** <http://www.yenka.com/freecontent/item.action?quick=ot#>

**Vertical shifting animation:** <http://www.yenka.com/freecontent/item.action?quick=ji#>

### Horizontal Transformations

To “see” horizontal transformations, the argument must be in factored form. Once in factored form, to compute both a horizontal stretch/flip/shrink and a horizontal shift on the same graph, FIRST stretch/flip/shrink, then shift. Remember that horizontal transformations are “opposite” of what they seem. It’s easiest to figure out the proper endpoints for one period. See below.

### Theorem on Phase Shifts and Periods

If  $y = \sin(bx + c)$  or  $y = \cos(bx + c)$  for any real numbers  $b$  and  $c$ ,  $b \neq 0$ , then

i) The period is  $\frac{2\pi}{|b|}$  and the phase shift is  $-\frac{c}{b}$ .

ii) The interval containing exactly one cycle can be found by solving the inequality  $0 \leq bx + c \leq 2\pi$ .

### Period Change Animation:

<http://www.mathwarehouse.com/trigonometry/period-sine-cosine/how-equation-effects-graph.php>

### Phase Shift Animation (:

[http://www.intmath.com/Trigonometric-graphs/3\\_Graphs-sin-cos-phase-shift.php](http://www.intmath.com/Trigonometric-graphs/3_Graphs-sin-cos-phase-shift.php)

1. Graph the equation.

$$y = 2 \sin(x) + 1$$

**Normal Sine Wave:**

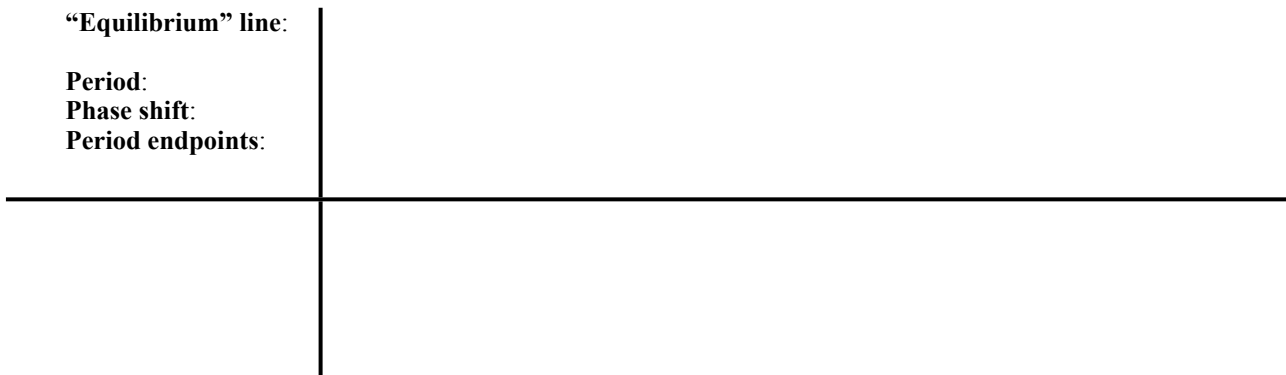
**Amplitude:**

**“Equilibrium” line:**

**Period:**

**Phase shift:**

**Period endpoints:**



2. *Graph the equation.*

$$y = \cos\left(2x + \frac{\pi}{2}\right)$$

**Normal Cosine Wave:**

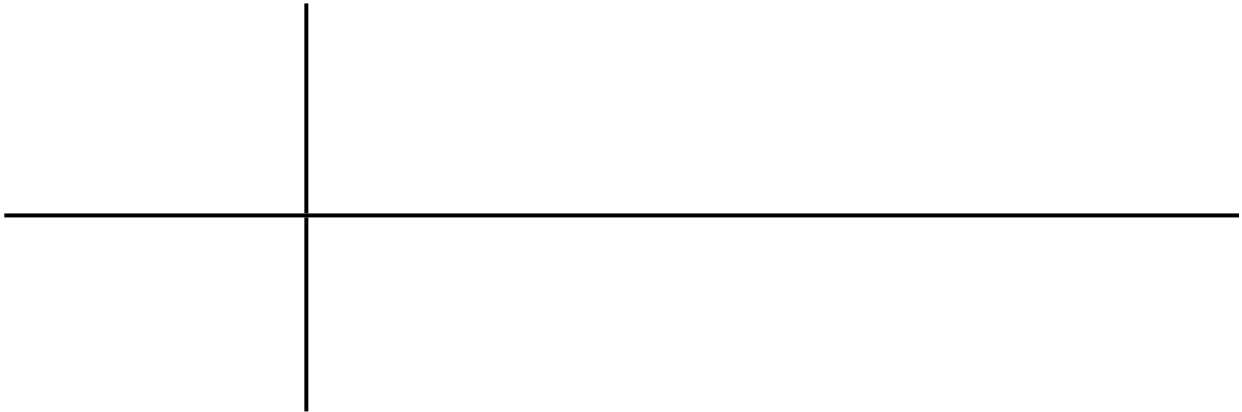
**Amplitude:**

**“Equilibrium” line:**

**Period:**

**Phase shift:**

**Period endpoints:**



3. *Graph the equation.*

$$y = -3\cos(x) - 2$$

**Normal Sine Wave:**

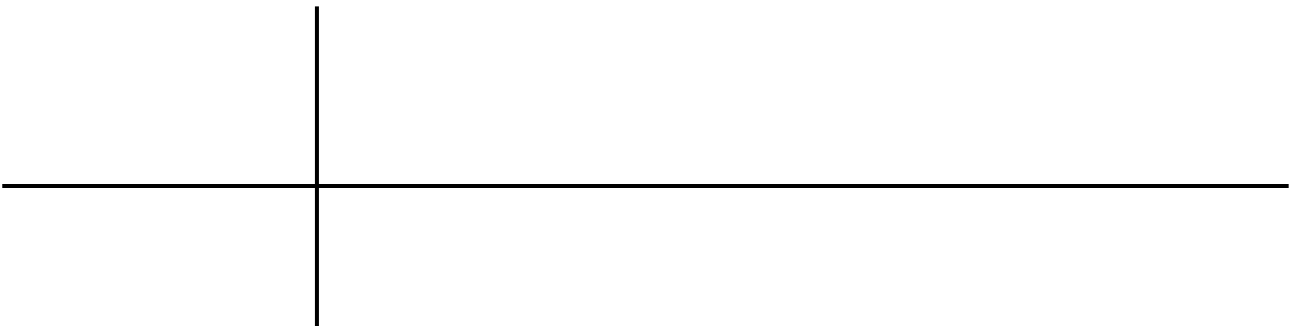
**Amplitude:**

**“Equilibrium” line:**

**Period:**

**Phase shift:**

**Period endpoints:**



4. *Graph the equation.*

$$y = -4 \cos\left(2x + \frac{\pi}{3}\right)$$

**Normal Cosine Wave:**

**Amplitude:**

**“Equilibrium” line:**

**Period:**

**Phase shift:**

**Period endpoints:**

