

## Transformations of Tangent, Cotangent, Cosecant and Secant — 7.7

### Theorem on the Graph of Tangent

If  $y = a \tan(bx + c) + d$  for any real numbers  $a, b, c$  and  $d$ ,  $a, b \neq 0$ , then

- The period is  $\frac{\pi}{|b|}$  and the phase shift is  $-\frac{c}{b}$ .
- Successive vertical asymptotes for the graph of one branch may be found by solving the inequality 
$$-\frac{\pi}{2} \leq bx + c \leq \frac{\pi}{2}.$$

1. Graph the equation.

$$y = \tan\left(2x - \frac{\pi}{2}\right) + 3$$

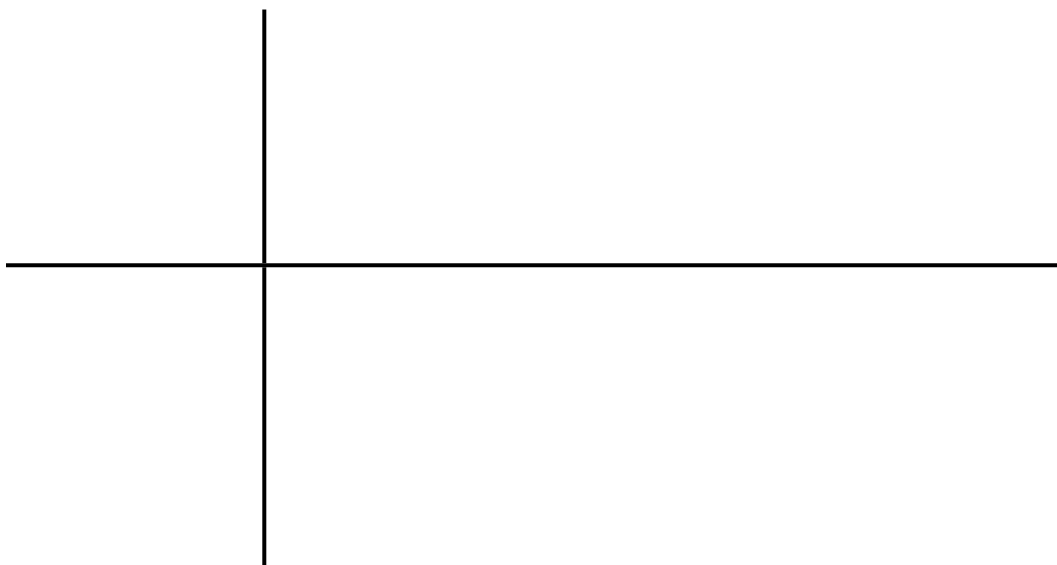
Normal Tangent Graph:

“Equilibrium” line:

Period:

Phase shift:

Asymptotes:



### Theorem on the Graph of Cotangent

If  $y = a \cot(bx + c) + d$  for any real numbers  $a, b, c$  and  $d$ ,  $a, b \neq 0$ , then

- The period is  $\frac{\pi}{|b|}$  and the phase shift is  $-\frac{c}{b}$ .
- Successive vertical asymptotes for the graph of one branch may be found by solving the inequality  $0 \leq bx + c \leq \pi$ .

2. Graph the equation.

$$y = -3 \cos(x) - 2$$

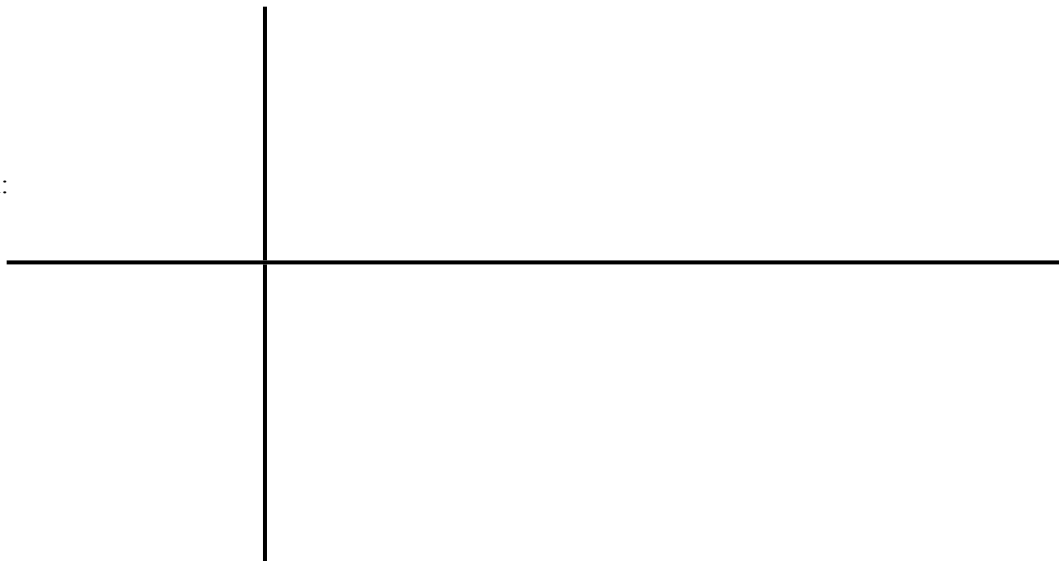
Normal Cotangent Graph:

“Equilibrium” line:

Period:

Phase shift:

Asymptotes:



### Theorem on the Graph of Cosecant and Secant

If  $y = a \csc(bx + c) + d$  or  $y = a \sec(bx + c) + d$  for any real numbers  $a$ ,  $b$ ,  $c$  and  $d$ ,  $a, b \neq 0$ , then

- i) The period is  $\frac{2\pi}{|b|}$  and the phase shift is  $-\frac{c}{b}$ .
- ii) To find the vertical asymptotes for a cosecant graph, sketch a graph of sine with the corresponding transformations. To find the vertical asymptotes for a secant graph, sketch a graph of cosine with the corresponding transformations.

3. Graph the equation.

$$y = -\frac{1}{2} \csc(2x - \pi)$$

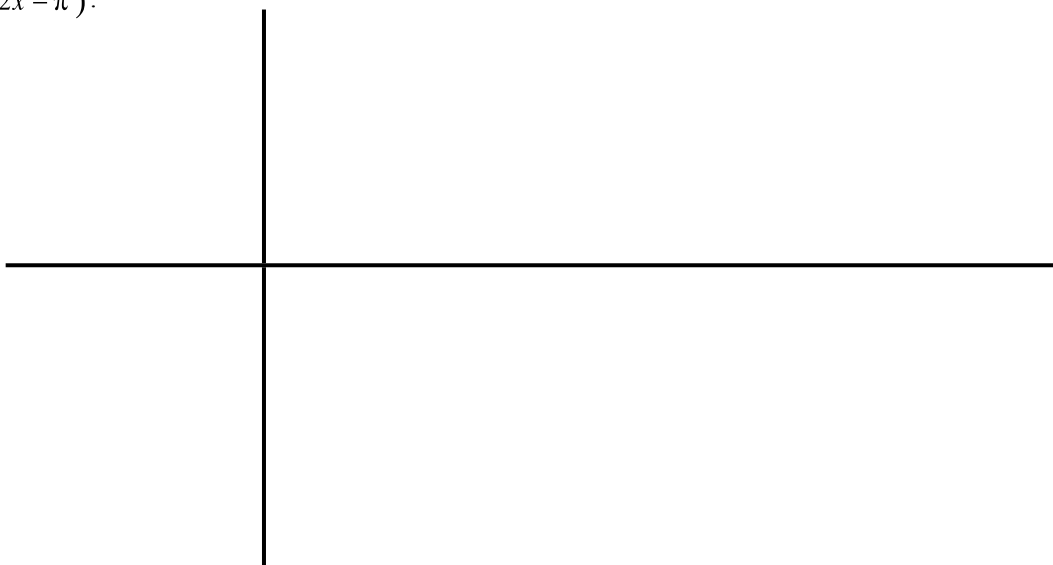
Graph of  $y = -\frac{1}{2} \sin(2x - \pi)$ :

“Equilibrium” line:

Period:

Phase shift:

Asymptotes:



4. Graph the equation.

$$y = \sec\left(\frac{\pi}{8}x\right) - 2$$

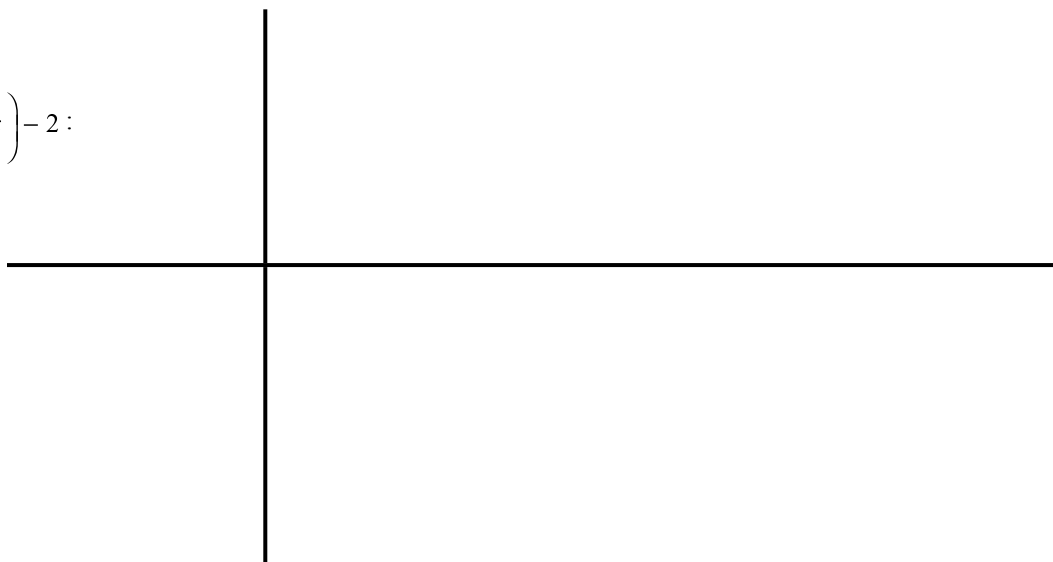
Graph of  $y = \cos\left(\frac{\pi}{8}x\right) - 2$ :

“Equilibrium” line:

Period:

Phase shift:

Asymptotes:



5. *Graph the equation.*

$$y = |\sin x| + 2$$

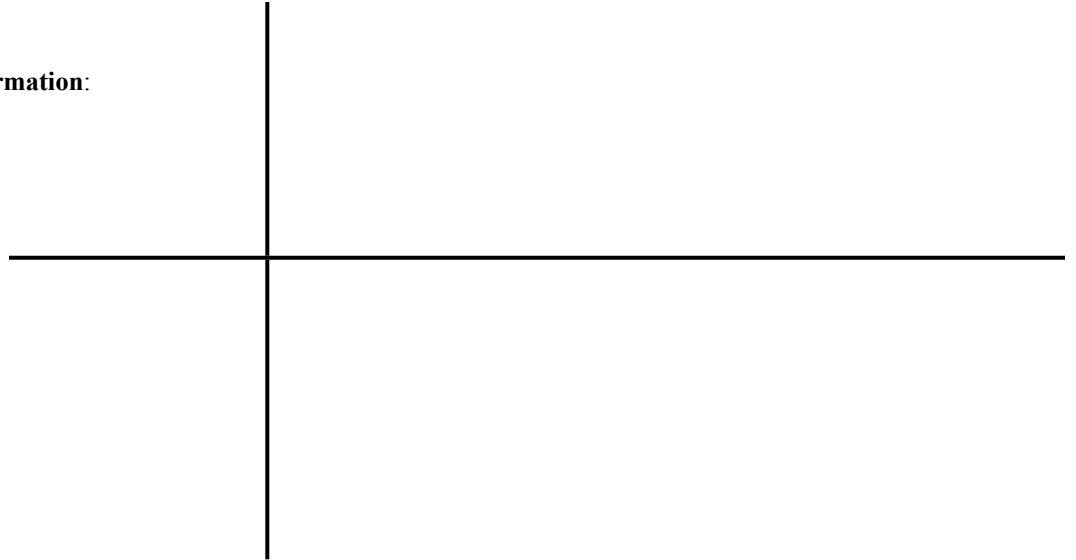
**Normal Sine Wave:**

**Amplitude:**

**Period:**

**Absolute value transformation:**

**“Equilibrium” line:**



6. *Graph the equation.*

$$y = |\cos(\pi x)|$$

**Normal Cosine Wave:**

**Amplitude:**

**Period:**

**Period endpoints:**

**Absolute value transformation:**

