Math 4 Pre-Calculus
Lecture 19, Part 1

Homework
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Inverse Trigonometric Functions — 8.1 and 8.2

Inverse Sine Function

The inverse sine function, denoted by $\sin^{-1} x$ or $\arcsin x$, is defined by

$$y = \sin^{-1} x \quad \text{if and only if} \quad x = \sin y$$
$$y = \arcsin x \quad \text{if and only if} \quad x = \sin y$$

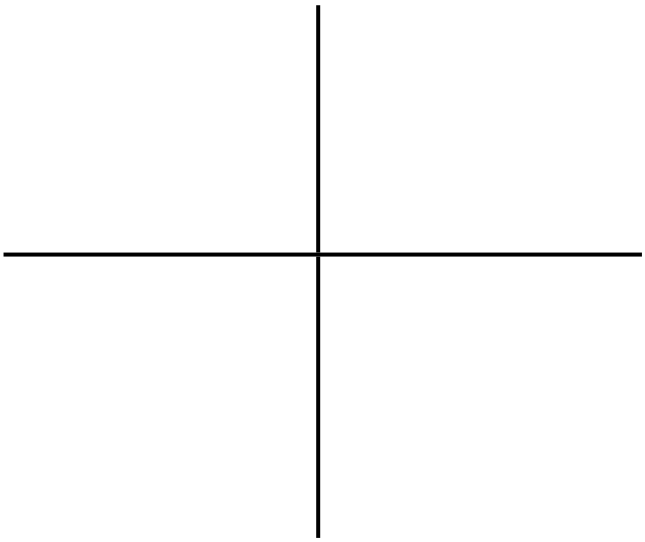
for $-1 \le x \le 1$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$. This limits the range to angles in Quadrants I and IV.

1. *Graph* $y = \sin^{-1} x = \arcsin x$.

Domain:

Range:

Symmetry:



Properties of Inverse Sine

$\sin\left(\sin^{-1} x\right) = \sin\left(\arcsin x\right) = x$ if $-1 \le x \le 1$

$\sin^{-1}\left(\sin y\right) = \arcsin\left(\sin y\right) = y$ if $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

2. $\sin\left(\sin^{-1}\frac{1}{2}\right)$

3. $\arcsin\left(\sin\frac{2\pi}{3}\right)$

Inverse Cosine Function

The inverse cosine function, denoted by $\cos^{-1} x$ or $\arccos x$, is defined by

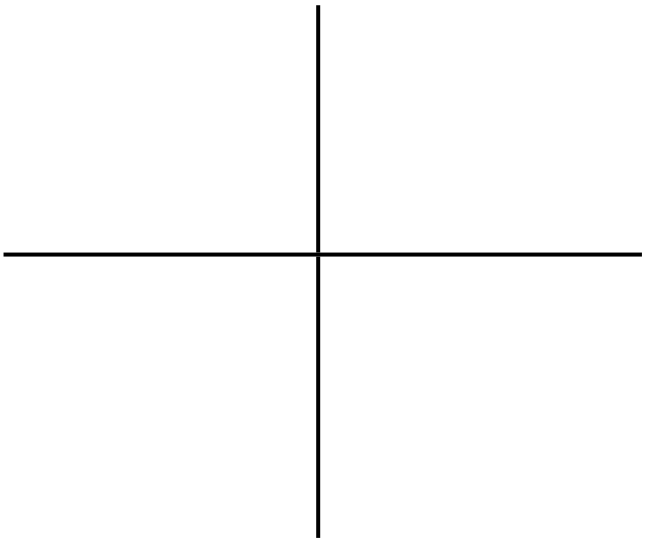
$$y = \cos^{-1} x \quad \text{if and only if} \quad x = \cos y$$
$$y = \arccos x \quad \text{if and only if} \quad x = \cos y$$

for $-1 \le x \le 1$ and $0 \le y \le \pi$. This limits the range to angles in Quadrants I and II.

4. *Graph* $y = \cos^{-1} x = \arccos x$.

Domain:

Range:



Properties of Inverse Cosine

$\cos\left(\cos^{-1} x\right) = \cos\left(\arccos x\right) = x$ if $-1 \le x \le 1$

$\cos^{-1}\left(\cos y\right) = \arccos\left(\cos y\right) = y$ if $0 \le y \le \pi$

5. $\cos\left[\cos^{-1}\left(-0.5\right)\right]$

6. $\arccos\left(\cos\pi\right)$

Inverse Tangent Function

The inverse tangent function, denoted by $\tan^{-1} x$ or $\arctan x$, is defined by

$$y = \tan^{-1} x \quad \text{if and only if} \quad x = \tan y$$
$$y = \arctan x \quad \text{if and only if} \quad x = \tan y$$

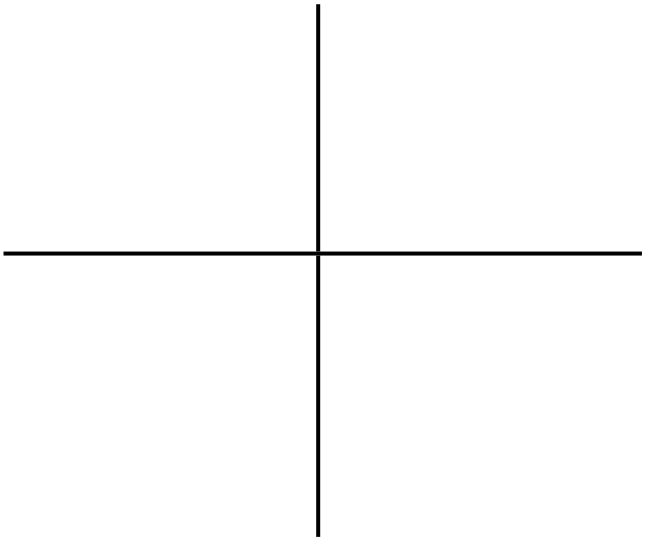
for any real number x and $-\frac{\pi}{2} < y < \frac{\pi}{2}$. This limits the range to angles in Quadrants I and IV.

7. *Graph* $y = \tan^{-1} x = \arctan x$.

Domain:

Range:

Horizontal Asymptotes:



Properties of Inverse Tangent

$\tan\left(\tan^{-1} x\right) = \tan\left(\arctan x\right) = x$ for every real number x

$\tan^{-1}\left(\tan y\right) = \arctan\left(\tan y\right) = y$ if $-\frac{\pi}{2} < y < \frac{\pi}{2}$

8. $\tan\left(\tan^{-1} 2500\right)$

9. $\arctan\left(\tan \pi\right)$

Find the exact value of the expression whenever it is defined.

10. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

11. $\tan\left(\cos^{-1} 0\right)$

12. $\cot\left(\sin^{-1}\left(-\frac{2}{5}\right)\right)$

13. $\cos\left(\frac{1}{2}\arctan\left(\frac{8}{15}\right)\right)$

Write the expression as an algebraic expression in x for $x > 0$.

14. $\tan(\arccos x)$

15. $\cot\left(\sin^{-1}\dfrac{\sqrt{x^2-9}}{x}\right)$

The given equation has the form $y = f(x)$ ***a)*** *Find the domain of* f ***b)*** *Find the range of* f ***c)*** *Solve for* x *in terms of* y.

16. $y = 2\sin^{-1}(3x - 4)$

Solve the equation for x *in terms of* y *if* x *is restricted to the given interval.*

17. $y = 2 + 3\sin x$ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Use inverse trigonometric functions to find the solutions of the equation that are in the given interval, and approximate the solutions to four decimal places.

18. $6\sin 2x - 8\cos x + 9\sin x - 6 = 0$ $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$