

## Multiple-Angle Formulas — 8.5

### Double-Angle Formulas

$$\begin{aligned}\sin(2u) &= 2 \sin u \cos u & \cos(2u) &= \cos^2 u - \sin^2 u & \tan(2u) &= \frac{2 \tan u}{1 - \tan^2 u} \\ \cos(2u) &= 1 - 2 \sin^2 u \\ \cos(2u) &= 2 \cos^2 u - 1\end{aligned}$$

Find the exact values of  $\sin 2\theta$ ,  $\cos 2\theta$ , and  $\tan 2\theta$  for the given values of  $\theta$ .

1.  $\sin \theta = -\frac{4}{5}; \quad 270^\circ < \theta < 360^\circ$

### Half-Angle Identities

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

### Half-Angle Formulas

$$\sin \frac{v}{2} = \pm \sqrt{\frac{1 - \cos v}{2}}$$

$$\cos \frac{v}{2} = \pm \sqrt{\frac{1 + \cos v}{2}}$$

$$\tan \frac{v}{2} = \pm \sqrt{\frac{1 - \cos v}{1 + \cos v}} = \frac{1 - \cos v}{\sin v} = \frac{\sin v}{1 + \cos v}$$

Find the exact values of  $\sin \frac{\theta}{2}$ ,  $\cos \frac{\theta}{2}$ , and  $\tan \frac{\theta}{2}$  for the given conditions.

2.  $\sec \theta = -4$ ;  $180^\circ < \theta < 270^\circ$

Use the half-angle formulas to find the exact values.

3.  $\sin 157^\circ 30'$

Verify the identity.

4.  $\csc 2u = \frac{1}{2} \csc u \sec u$

5.  $\tan^2 \frac{\theta}{2} = 1 - 2 \cot \theta \csc \theta + 2 \cot^2 \theta$

6.  $\cos^4 x - \sin^4 x = \cos 2x$

*Find the solutions of the equation that are in the interval  $[0, 2\pi)$ .*

7.  $\tan 2t - 2 \cos t = 0$