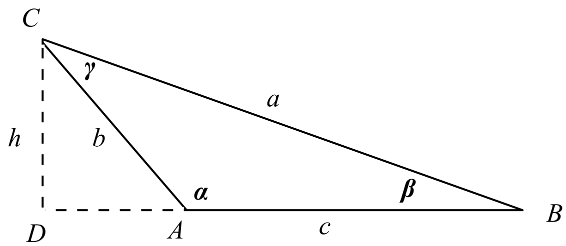
Math 4 Pre-Calculus

Homework
pg. 682: 9 – 33eoo, 37, 41, 42, 45 47
pg. 689: 9 – 29eoo, 35, 36, 39, 45

Law of Sines & Law of Cosines — 9.2 & 9.3

Labeling an Oblique Triangle

An oblique triangle is a triangle that does not contain a right angle. ΔABC is an oblique triangle. We've been working strictly with right triangles like ΔDBC.



Law of Sines

If ABC is an oblique triangle labeled in the usual manner then

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

You can use the law of sines to solve oblique triangles in either of the following cases:

1. Two sides and an angle *opposite* one of them is given. (SSA)
2. Two angles and any side are given. (AAS or ASA)

Note: if you have SAS (two sides and the angle between them) or SSS (all 3 sides but no angle), a law called **Law of Cosines** is used (discussed later in these notes)

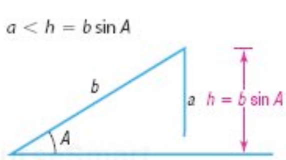
Solve ΔABC.

1. $\alpha = 48°$, $\gamma = 57°$, and $b = 47$

Possible Oblique Triangles

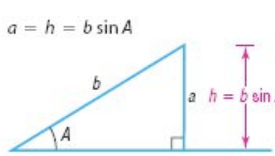
ABC is an oblique triangle labeled in the usual manner and b is a given length and α is a given angle. There are four possible outcomes illustrated below:

No Triangle If $a < h = b \sin A$, then side a is not sufficiently long to form a triangle. See Figure 14.



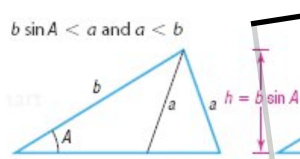
Case 1:
No triangle is formed.

One Right Triangle If $a = h = b \sin A$, then side a is just long enough to form a right triangle. See Figure 15.



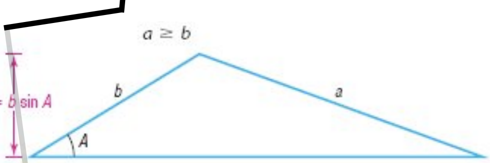
Case 2:
A right triangle is formed.

Two Triangles If $h = b \sin A < a$, and $a < b$ two distinct triangles can be formed from the given information. See Figure 16.



Case 3:
Two triangles are formed.

One Triangle If $a \geq b$, only one triangle can be formed. See Figure 17.



Case 4:
One oblique triangle is formed.

Solve ΔABC.

2. $\alpha = 67°$, $a = 100$, and $c = 125$

3. $a = 12.4$, $b = 8.7$, and $\beta = 36.7°$

4. To determine the distance between two points *A* and *B*, a surveyor chooses a point *C* that is 375 yards from *A* and 530 yards from *B*. If $\angle BAC$ has measure 49.5°, approximate the distance between *A* and *B*. *Give your answer accurate to the nearest yard.*

5. Shown in the figure is a solar panel 10 feet in width, which is to be attached to a roof that makes an angle of 25° with the horizontal. Approximate the length d of the brace that is needed for the panel to make an angle of 45° with the horizontal. *Give your answer accurate to two decimal places.*

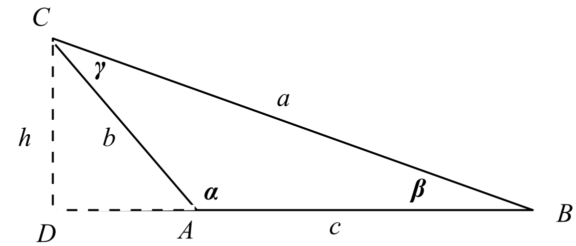
6. A helicopter hovers at an altitude that is 1000 feet above a mountain peak of altitude 5210 feet, as shown in the figure. A second, taller peak is viewed from both the mountaintop and the helicopter. From the helicopter, the angle of depression is 43°, and from the mountaintop, the angle of elevation is 18°.

   a) Find the distance from peak to peak. *Give your answer accurate to the nearest foot.*

   b) Find the altitude of the higher peak. *Give your answer accurate to the nearest foot.*

Law of Cosines

If ABC is a triangle labeled as in the illustration then



1. $a^2 = b^2 + c^2 - 2bc\cos\alpha$
2. $b^2 = a^2 + c^2 - 2ac\cos\beta$
3. $c^2 = a^2 + b^2 - 2ab\cos\gamma$

You can use the law of cosines to solve triangles in for either of the following cases:

1. Two sides and an angle *between* them is given. (SAS)
2. Three sides are given (SSS). *If this is the case, always find the largest angle first. The largest angle is the angle opposite the longest side.*

Solve ΔABC.

7. $a = 5.0$, $c = 8.0$, and $\beta = 77°$

8. $a = 90$, $b = 70$, and $c = 40$

9. A vertical pole 40 feet tall stands on a hillside that makes an angle of $17°$ with the horizontal. Approximate the minimal length of cable that will reach from the top of the pole to a point 72 feet downhill from the base of the pole. *Give your answer accurate to the nearest foot.*

10. Pick three things (buildings, trees, poles, etc.) on campus, where you are able to walk in a straight line from each one to another. Count off the number of "paces" from one to the other, then use the law of cosines to determine the angle that you walked to get from one to another..