Math 4 Pre-Calculus

Homework
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Vectors — 10.4

Definitions

Scalar quantities are quantities (such as length, area, volume, time or temperature) that only have a magnitude, not a direction.

Vectors are quantities (such as velocity or force) that have both a magnitude and a direction. Vectors can be thought of as directed line segments.

The vector $\overrightarrow{PQ}$ has an initial point P and a terminal point Q.

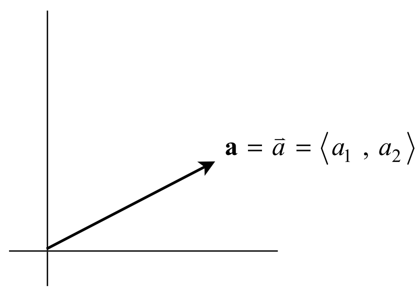
The magnitude of $\overrightarrow{PQ}$, denoted by $\left\| \overrightarrow{PQ} \right\|$ is the length of the line segment.

Vectors may also be written in bold letters, such as $\mathbf{u}$ or $\mathbf{v}$, or handwritten as $\vec{u}$ or $\vec{v}$.

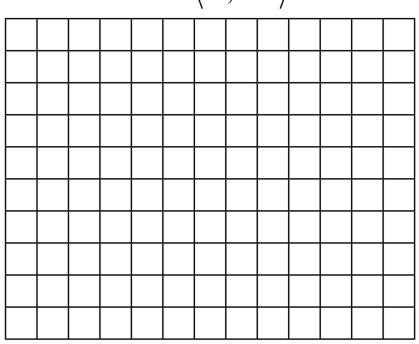
Note: Vectors with the same magnitude and the same direction are equivalent.

Vectors in the XY-Plane

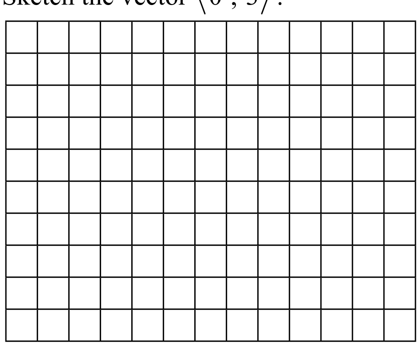
The <u>position vector</u> $\mathbf{a}$ in the xy-plane with an initial point at the origin $(0\ ,\ 0)$ and a terminal point at $(a_1\ ,\ a_2)$ is denoted by $\mathbf{a} = \vec{a} = \langle a_1\ ,\ a_2 \rangle$. See illustration below.



1. Sketch the vector $\langle 3\ ,\ -5 \rangle$.



Sketch the vector $\langle 0\ ,\ 3 \rangle$.



2. What is the magnitude of the vector $\langle 0 , 3 \rangle$?

The Magnitude of a Vector

The magnitude of a vector $\mathbf{a} = \vec{a} = \langle a_1 , a_2 \rangle$, denoted by $\|\vec{a}\|$ is given by $\|\vec{a}\| = \|\langle a_1 , a_2 \rangle\| = \sqrt{a_1^{\,2} + a_2^{\,2}}$.

3. What is the magnitude of the vector $\langle 3 , -5 \rangle$?

4. Find the magnitude of $\langle \frac{1}{2} , \frac{5}{4} \rangle$.

Addition of Vectors

$$\mathbf{a} + \mathbf{b} = \langle a_1 , a_2 \rangle + \langle b_1 , b_2 \rangle = \langle a_1 + b_1 , a_2 + b_2 \rangle$$

Find the following.

5. $\langle 3\ ,\ 2\rangle + \langle 1\ ,\ 5\rangle$

6. $\langle 0\ ,\ 7\rangle + \langle -3\ ,\ 4\rangle$

Scalar Multiplication

$$m\langle a_1\ ,\ a_2\rangle = \langle m\,a_1\ ,\ m\,a_2\rangle$$

7. If $\mathbf{a} = \langle 1\ ,\ 3\rangle$, find $5\mathbf{a}$ and $-2\mathbf{a}$.

Definitions

$$\mathbf{0} = \langle 0\ ,\ 0\rangle \qquad -\mathbf{a} = -\langle a_1\ ,\ a_2\rangle = \langle -a_1\ ,\ -a_2\rangle$$

Vector Subtraction

$$\mathbf{a} - \mathbf{b} = \langle a_1\ ,\ a_2\rangle - \langle b_1\ ,\ b_2\rangle = \langle a_1 - b_1\ ,\ a_2 - b_2\rangle$$

8. If $\mathbf{a} = \langle 8\ ,\ -1\rangle$ and $\mathbf{b} = \langle -2\ ,\ 5\rangle$, find $\mathbf{a} - \mathbf{b}$.

Unit Vectors: i and j

$\mathbf{i} = \langle 1 , 0 \rangle$ $\mathbf{j} = \langle 0 , 1 \rangle$

Both $\mathbf{i}$ and $\mathbf{j}$ are unit vectors; in other words their magnitudes are 1.

Alternate way of writing vectors:

If $\mathbf{a} = \langle a_1 , a_2 \rangle$ then

$$\mathbf{a} = \langle a_1 , 0 \rangle + \langle 0 , a_2 \rangle$$
$$= a_1 \langle 1 , 0 \rangle + a_2 \langle 0 , 1 \rangle$$
$$= a_1\mathbf{i} + a_2\mathbf{j}$$

So, the $\mathbf{i}\ \mathbf{j}$ form for vectors is: $\mathbf{a} = \langle a_1 , a_2 \rangle = a_1\mathbf{i} + a_2\mathbf{j}$

or handwritten $\vec{a} = \langle a_1 , a_2 \rangle = a_1\vec{i} + a_2\vec{j}$

Write each of the following in $\mathbf{i}\ \mathbf{j}$ form.

9. $\langle 5 , 3 \rangle$

10. $\langle 4 , -1 \rangle$

11. $\langle 0 , 3 \rangle$

12. If $\mathbf{a} = 5\mathbf{i} + \mathbf{j}$ and $\mathbf{b} = 4\mathbf{i} - 7\mathbf{j}$, express $3\mathbf{a} - 2\mathbf{b}$ as a linear combination of $\mathbf{i}$ and $\mathbf{j}$.

Formulas for Horizontal and Vertical Components of Vectors

If the vector $\mathbf{a} = \langle a_1 , a_2 \rangle$ and the angle θ is in standard position (measured from the positive x-axis) as shown, then

$$a_1 = \|a\| \cos\theta , \qquad \text{and} \qquad a_2 = \|a\| \sin\theta .$$

Find the magnitude of the vector $\mathbf{a}$ *and the smallest positive angle* θ *from the positive x-axis to the vector* $\overrightarrow{OP}$ *that corresponds to* $\mathbf{a}$.

13. $\mathbf{a} = \langle -2 , -2\sqrt{3} \rangle$

14. $\mathbf{a} = 10\mathbf{i} - 10\mathbf{j}$