Math 4 Pre-Calculus
Summer 2009

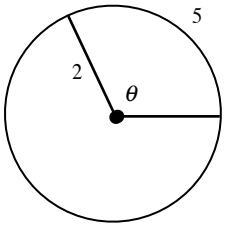
Name_____
Date_____

Practice Exam 3

No Graphing calculators used on the exam. Scientific calculators without symbol manipulation capabilities may be used.
Remember to check solutions when solving equations if necessary. Simplify answers as much as possible.
Draw pictures whenever possible.

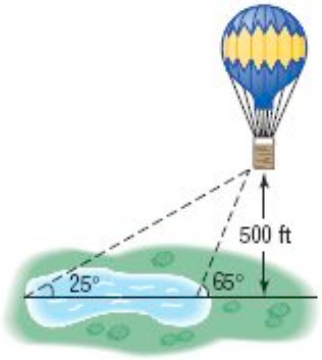
1. Angle Conversions:
   a. Convert the angle $41^\circ 20' 35''$ to a decimal in degrees.
      Round the answer to two decimal places.
   b. Convert the angle 63.18° to degrees minutes and seconds.

2. Arc Length and Area of a Sector:
   Find the area of the sector having this angle.



3. Find the values of the remaining trigonometric functions given that $\tan\theta = \frac{15}{5}$ and $\sin\theta < 0$.

4. From a stationary hot-air balloon 500 feet above the ground, two sightings of a lake are made. How long is the lake.



5. For the function $f(x) = 2\cos\left(2x + \frac{\pi}{3}\right) + 1$

Find the
   a. Amplitude
   b. Period
   c. Horizontal (Phase) Shift
   d. Vertical Shift
   e. Domain
   f. Range

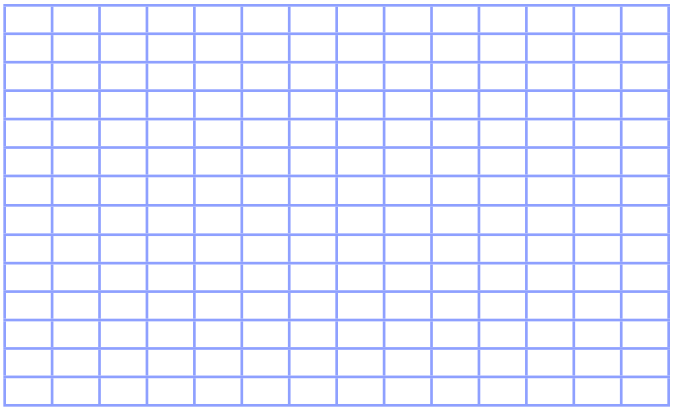
and then graph at least two periods of it.

6. For the function $f(x) = -\tan(3x)$

find the

a. Amplitude
b. Period
c. Horizontal (Phase) Shift
d. Vertical Shift
e. Domain
f. Range

and then graph at least two periods of it.



7. Composing trig functions and inverse trig functions without calculators:

a. Find the exact value of $\sin^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right)$.

b. Find the exact value of $\tan\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$.

8. Find exact solutions to the equation given below:

$$3\sin^{-1}(x) = \pi$$

9. Write the trigonometric expression

$$\cos\left(\csc^{-1} x\right)$$

as a expression in terms of x (without the inverse trig and trig functions).

10. Verify the following identity using the methods we've discussed in class.

$$\sec\theta = \sin\theta\,(\tan\theta + \cot\theta)$$

11. Verify the following identity

$$\cos(\alpha+\beta)\cos(\alpha-\beta) = \cos^2(\alpha) - \sin^2(\beta)$$

12. Given that $\sin\alpha = \frac{4}{5}$ where $0 < \alpha < \frac{\pi}{2}$ and $\sin\beta = \frac{5}{13}$ where $\frac{\pi}{2} < \beta < \pi$

Find the following:

a) $\sin(2\alpha)$

b) $\cos(\alpha-\beta)$

13. Prove that an isosceles right triangle can have legs whose lengths are equal to $\frac{\sqrt{2}}{2}$. Include a picture of the labelled triangle with your work.

14. Prove that a $30^\circ\ 60^\circ\ 90^\circ$, triangle can have legs whose lengths are equal to $\frac{1}{2}$ and $\frac{\sqrt{3}}{2}$. Include a picture of the labelled triangle with your work.

15. Why is $\sin\theta$ negative for some values of θ greater than 90° and what are they.

16. Show geometrically using a unit circle why $\cos\theta$ is an even function and why $\sin\theta$ is an odd function.

17. How many degrees do you have to rotate an incandescent light bulb to screw it into a light socket? (and how many mathematicians would it take to screw it in? Answer for second part of question is below….don't look.)