6856320`Polynomial Functions

Global Behavior		**Degree of Polynomial**	
		Odd	**Even**
Leading Coefficient	**Positive**	as $x \to \infty$, $f(x) \to \infty$ AND as $x \to -\infty$, $f(x) \to -\infty$	as $x \to \infty$, $f(x) \to \infty$ AND as $x \to -\infty$, $f(x) \to \infty$
		Draw a graph with above characteristics:	Draw a graph with above characteristics:
	Negative	as $x \to \infty$, $f(x) \to -\infty$ AND as $x \to -\infty$, $f(x) \to \infty$	as $x \to \infty$, $f(x) \to -\infty$ AND as $x \to -\infty$, $f(x) \to -\infty$
		Draw a graph with above characteristics:	Draw a graph with above characteristics:

Unlike numbers, some functions are neither even nor odd. But often, in Calculus, we can take advantage of symmetry of even and odd functions to simplify calculations if we recognize that we have such a function.

Determine whether the following functions are even, odd or neither.

5. a. $f(x) = x^3 - x$ b. $f(x) = (x-1)^2$ c. $f(x) = \cos x$

Roots of Polynomials

To find the roots of a polynomial, try factoring, long division or synthetic division. The **real** roots of polynomials correspond to x-intercepts. Once the x-intercepts of a polynomial function are found, use a sign chart (or the root multiplicities) to determine the behavior of the function near each x–intercept. If the multiplicity is even, the graph behaves the same on both sides of the x–intercept (bounces off x–intercept; i.e. behaves the same on either side of x–intercept). If the multiplicity is odd, the graph behaves the opposite on both sides of the x–intercept (crosses x–intercept; i.e. behaves opposite on either side of x–intercept).

Sketch a graph of each of the following.

1. $f(x) = -x(x+1)^2$

2. $g(x) = x^2(x-2)^2(x+1)$

3. $h(x) = x^4 + 4x^3 + 3x^2 - 4x - 4$ (hint: -2 is a root)

Rational Functions

A function is a **rational function** if $f(x) = \dfrac{g(x)}{h(x)}$, where $g(x)$ and $h(x)$ are polynomials. All values that make the denominator $h(x)$ equal to zero are excluded from the **domain**. That is domain $f(x) = \{x | h(x) \neq 0\}$

Vertical asymptotes occur at x–values for which the denominator $h(x) = 0$ but the numerator $g(x) \neq 0$.

Holes on the graph can occur at x–values for which both the denominator $h(x) = 0$ and the numerator $g(x) = 0$.

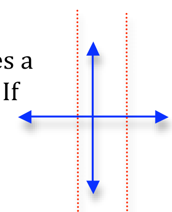
X–intercepts occur where a x-values for which the numerator $g(x) = 0$ but the denominator $h(x) \neq 0$.

Non-vertical asymptotes depend on the degrees of the numerator $g(x)$ and denominator $h(x)$

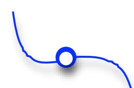
The **range** depends on the global behavior which can sometimes be determined by these asymptotes.

Notes on Rational Function behavior

Vertical asymptotes: The graph will ***never*** cross a vertical asymptote. The multiplicity of the factor that causes a vertical asymptote (in a completely reduced polynomial) determines the behavior of graph on either side of it. If the multiplicity is even the graph behaves the same on both sides. If the multiplicity is odd, the graph behaves opposite on both sides.



Holes: a hole is a location on the graph in the xy-coordinate system so it's got a y–coordinate as well. To find it, plug the x–value into the completely reduced rational function.



X–intercepts: occur when $f(x) = 0$. Being a fraction, $\dfrac{g(x)}{h(x)} = 0$ and the only time it can be zero is when the numerator, $g(x) = 0$ and the denominator, $h(x) \neq 0$.

Non-vertical asymptotes: the graph may cross a non–vertical asymptote and you should check to see if it does and if so, where. To find out where, set the function equal to the non-vertical asymptote and solve for x. As for determining what *kind* of non-vertical asymptote the function has, you can EITHER do long division

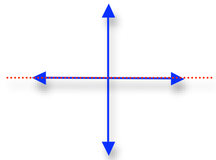
$$\begin{array}{r} q(x) \\ h(x)\overline{)g(x)} \\ \vdots \\ \overline{r(x)} \end{array}$$

so that $\dfrac{g(x)}{h(x)} = q(x) + \dfrac{r(x)}{h(x)}$ and $y = q(x)$ will be the non-vertical asymptote.

OR you can memorize the following:

Compare the degrees of the numerator and denominator.

if *Degree numerator < degree denominator*, the non-vertical asymptote is $y = 0$, a horizontal aymptote.

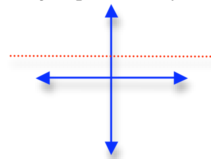


Note: $y = 0$ is a polynomial of degree 0

if *Degree numerator = degree denominator*, the non-vertical asymptote is $y = \dfrac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$

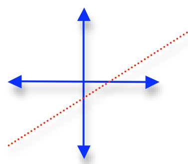
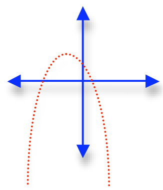
also a horizontal asymptote but not on the x-axis.

Note: $y = \dfrac{a}{b}$ which is a number or polynomial of degree 0



if *Degree numerator > degree denominator*, the non-vertical asymptote is not a horizontal asymptote. It is instead an oblique asymptote determined by long division as described above.

Note: $y = q(x)$ so the non-vertical asymptote is a polynomial of degree > 0

 or  or any other polynomial shape

All asymptotes and holes must be drawn when grapghing a function whose graph has them. If a function crosses a non-vertical asymptote, that point must be indicated.

Sketch a graph of each of the following.

1. $y = \dfrac{1}{x+3}$

Domain:

Holes:

Vertical Asymptote(s):

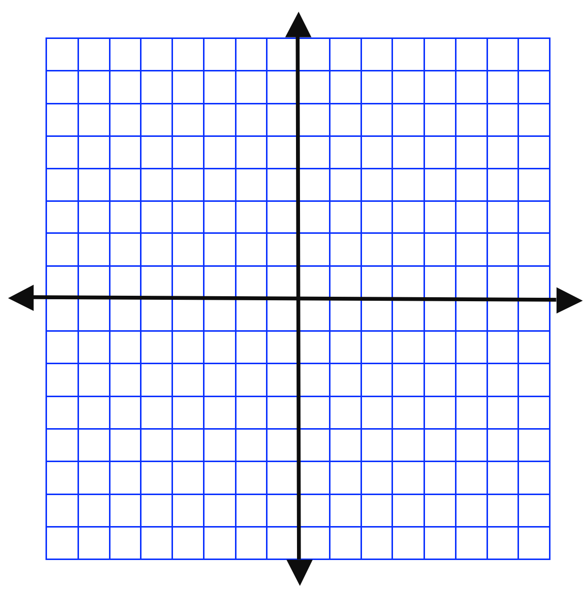
x–intercepts:

y–intercepts:

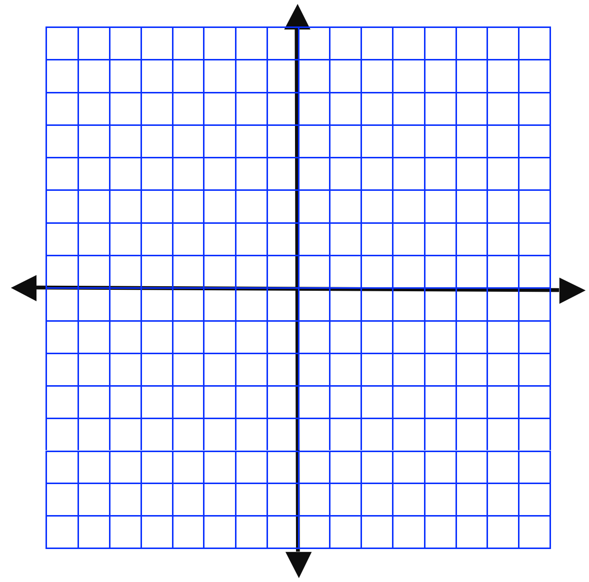
degree of numerator:
degree of denominator:

Non-vertical asymptote(s):

Does graph cross the non-vertical asymptote?



2. $\dfrac{x+2}{x^2+5x+6}$



Domain:

Holes:

Vertical Asymptote(s):

x–intercepts:

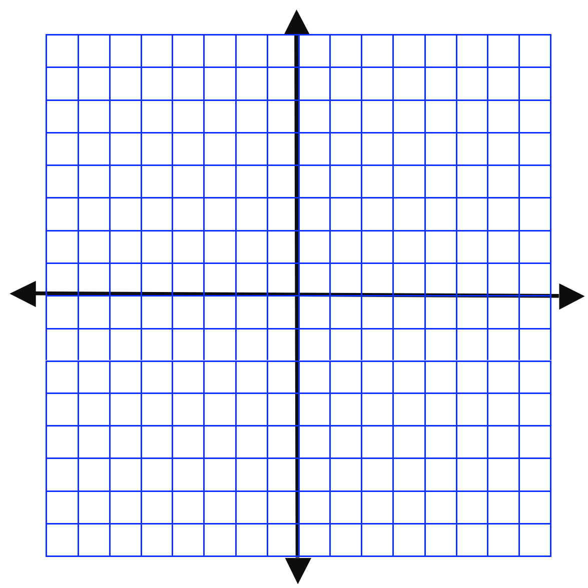
y–intercepts:

degree of numerator:
degree of denominator:

Non-vertical asymptote(s):

Does graph cross the non-vertical asymptote?

3. $y=\dfrac{x+5}{x+3}$



Domain:

Holes:

Vertical Asymptote(s):

x–intercepts:

y–intercepts:

degree of numerator:
degree of denominator:

Non-vertical asymptote(s):

Does graph cross the non-vertical asymptote?

4. $y = \dfrac{x^2 + 2x}{x - 3}$

Domain:

Holes:

Vertical Asymptote(s):

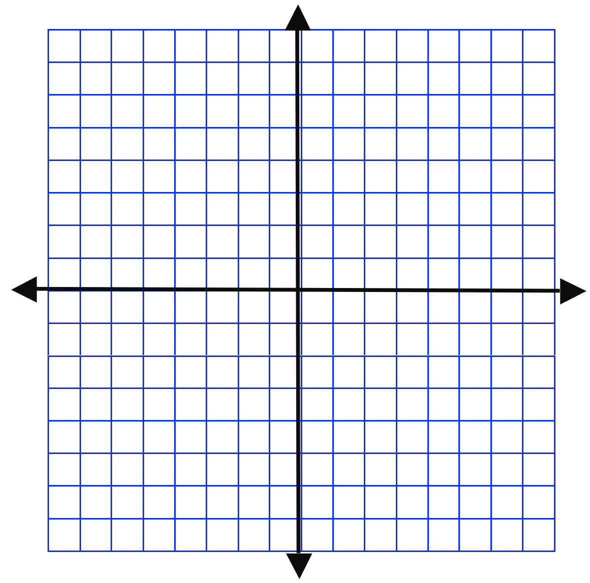
x–intercepts:

y–intercepts:

degree of numerator:
degree of denominator:

Non-vertical asymptote(s):

Does graph cross the non-vertical asymptote?



Rational Function Homework problems:
(To be turned in with first homework set – Please complete your work on another sheet of paper.)

For the following three rational functions find:

Domain:
Holes:
Vertical Asymptote(s):
x–intercepts:
y–intercepts:
degree of numerator:
degree of denominator:
Non-vertical asymptote(s):
Does graph cross the non-vertical asymptote?

And then graph the function including and labelling above findings.
Be sure to graph each on it's own coordinate system.

1. $u(t) = \dfrac{(t-2)^2}{t^2 - 25}$

2. $y = \dfrac{x^2 + 2x - 15}{(x+1)^2 (x-3)}$

3. $p(x) = \dfrac{x^2 + 4x}{x + 1}$

4.

$f(x) = \dfrac{x^2}{x^2 + 9}$