Infinite Limits

Infinite limits describe the behaviour of a function as $x \to a$. Infinite limits are equal to ∞ or $-\infty$. These limits describe the behaviour near a specific a value.

Example:

$$\lim_{x\to 1} \frac{1}{(x-1)^2}$$

Limits at Infinity

Limits at infinity describe the behaviour of a function as $x \to \infty$ (or $x \to -\infty$). Limits at infinity may equal a constant value or $\pm\infty$. These limits describe the global behaviour of a function; in other words, what happens at the "ends" of the graph.

Example:

$$\lim_{x\to \infty} \frac{1}{(x-1)^2}$$

Note: The limit laws may apply for limits at infinity but cannot be used for infinite limits.

Limits at Infinity Definition

Let f be a function that is defined on some interval (a,∞) or $(-\infty,a)$.

Then $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$

means that values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large (or large negatively).

Horizontal Asymptotes

The horizontal line $y = L$ is called a horizontal asymptote of a function f, if either

$$\lim_{x\to\infty} f(x) = L \qquad \lim_{x\to-\infty} f(x) = L$$

Find the horizontal asymptotes for the following functions (problems 1, 2, and 3) by using limits.

1. $f(x) = \dfrac{1}{x^{2n}}$ where $n \in \mathbb{N}$ (the natural numbers = {1, 2, 3, 4,…})

2. $f(x) = \tan^{-1} x$

3. $f(x) = e^x$

Methods for Finding Limits in Section 2.6

- Use limit laws for limits that exist and don't equal to ∞ or $-\infty$.
- Use direct substitution for continuous functions, where a (which is a specific value) is in the domain.
- Algebraic tools:
  - a) Factoring
  - b) Combining (adding/subtracting) rational expressions
  - c) Rationalizing the numerator or denominator
  - d) Replacing an absolute value expression with an appropriate non-absolute value expression
  - e) Dividing each term in a fraction by the highest power of the variable–be careful with square root factors!

Remember: $\sqrt{x^2} = |x| = \begin{cases} x & \text{if x} \geq 0 \\ -x & \text{if x} < 0 \end{cases}$

- Estimating by determining what happens algebraically.
  In general, it's important to know that:
  $\dfrac{\text{nonzero constant}}{\text{denominator approaching } 0} \to \infty$ (or $-\infty$ depending on the signs of the numerator and denominator)
  $\dfrac{\text{nonzero constant}}{\text{denominator approaching } \pm\infty} \to 0$ (either from above or below depending on the signs of numerator and denominator)
- Support your answer by thinking about the graph.

Find the limit. Do not use tables. State what each limit means graphically.

4. $\lim\limits_{x \to -\infty} \left(x^2 + x\right)$

5. $\lim_{x\to\infty} \frac{2-3x^2}{5x^2+4x}$

6. $\lim_{x\to\infty} \frac{1-e^{2x}}{e^x}$

7. $\lim_{x\to\infty} \frac{x+2}{\sqrt{9x^2+1}}$

8. $\lim_{x\to-\infty} \left(x+\sqrt{x^2+ax}\right)$

Use limits to find the horizontal and vertical asymptotes of each curve, and then sketch a graph of the function.

9. $f(x) = \dfrac{1+x^4}{x^2 - x^4}$