Explicitly Defined Functions

Over the past several sections, we have been differentiating equations of the form $y = f(x)$, where y is defined explicitly in terms of x. Explicit functions are the type of relationships that we have come to know as functions; in other words, these relationships pass the vertical line test. This means that the equation is written (or can be written) where y is defined solely in terms of x.

Examples: $y = x^2 + 9$ $y = e^x \sin x$ $y + 2x^5 = 10$

Implicitly Defined Functions

In this section, we will differentiate equations where y is defined implicitly in terms of x. Implicit functions are the type of equations that cannot be written so that y is defined solely in terms of x. In other words, it takes more than one function $y = f(x)$ to describe the relationship between x and y.

Examples: $y^2 = x^2 + 9$ $xy^2 = ye^x + 3$ $xy + y^3x^5 = 10$ $y = -\frac{x}{y}$

Example

Let's take a look at the implicit function: $x^2 + y^2 = 29$. This equation is a circle with a center of $(0, 0)$ and a radius of $\sqrt{29}$.

The two explicit functions that define the implicit function $x^2 + y^2 = 29$ can be found by solving the equation for y:

$y = f(x) = \sqrt{29 - x^2}$ and $y = g(x) = -\sqrt{29 - x^2}$

$f(x)$ represents the upper half of the circle and $g(x)$ represents the lower half of the circle. Since you need 2 functions of x to describe the equation $x^2 + y^2 = 29$, this equation is an implicit function.

1. Find $\frac{dy}{dx}$ at the point $(5, 2)$ for the curve $x^2 + y^2 = 29$ by figuring out on which half of the circle the point lies.

2. Find the equation of the tangent line to the curve $x^2 + y^2 = 29$ at the point $(5, 2)$.

Implicit Differentiation

Differentiate both sides of the equation with respect to x and then solve the resulting equatin for y'. Remember that y is a function of x so use the *Chain Rule* for any y's.

3. Find $\frac{dy}{dx}$ at the point $(5, 2)$ for the curve $x^2 + y^2 = 29$ by implicit differentiation.

4. Find $\frac{dy}{dx}$ for $y^3 + xy = 5$ by implicit differentiation.

5. Given $2y^3 + x^2 y - xy^3 = 2$, find $\frac{dy}{dx}$ (or y') by implicit differentiation

6. Given $e^y = xy$, find $\frac{dy}{dx}$ (or y') by implicit differentiation

7. Given $1 + x = \sin\left(xy^2\right)$, find $\frac{dy}{dx}$ (or y') by implicit differentiation

8. If $f(x) + x\cos\left(f(x)\right) = x^2$ and $f(0) = 3$, find $f'(0)$ (*the slope of the tangent line at* $(0\ ,3)$)