

1. Find  $\frac{dy}{dx}$  of  $y = \log_a x$  by first rewriting the logarithm in exponential form and then using implicit differentiation.

### Derivative of a Logarithmic Function

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

### Derivative of a Natural Logarithmic Function

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

### Algebra Laws of Logarithms

$$\begin{aligned}\log_a(xy) &= \log_a(x) + \log_a(y) \\ \log_a\left(\frac{x}{y}\right) &= \log_a(x) - \log_a(y) \\ \log_a(x^r) &= r \log_a(x)\end{aligned}$$

2. Differentiate using the laws of logarithms.  $f(x) = \ln\left(\frac{x+1}{\sqrt{x-3}}\right)$

3. Find  $f'(x)$  if  $f(x) = \ln|x|$ .

$$\boxed{\frac{d}{dx} \ln|x| = \frac{1}{x}}$$

*Differentiate  $f$  and find the domain of  $f$*

4.  $f(x) = \ln(x^2 - 2x)$

*Find an equation of the tangent line to the curve at the given point*

5.  $y = \ln(x^2 - 3)$        $(2, 0)$

**Logarithmic Differentiation**

Use logarithmic differentiation to take the derivatives of functions where the exponent is also a function—any function in the form:  $y = f(x)^{g(x)}$ .

1. Take the natural logarithm of both sides of an equation  $y = f(x)^{g(x)}$  and use the Laws of Logarithms to simplify and “bring down” the power  $g(x)$ .
2. Differentiate implicitly with respect to  $x$ .
3. Solve the resulting equation for  $y'$ .

*Use logarithmic differentiation to find the derivative of the function.*

6.  $y = (\sqrt{x})^x$

7.  $y = x^{\cos x}$

**Derivatives of Inverse Trig Functions**

1. Find  $\frac{dy}{dx}$  for  $y = \sin^{-1} x$

2. Show that  $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

**Derivatives of Inverse Trigonometric Functions**

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$$

Note: the formulas for the derivatives of  $\csc^{-1} x$  and  $\sec^{-1} x$  depend on the domain of these fcns

3. Find the derivatives of the function  $y = \sqrt{x^2 - 1} \cdot \sec^{-1}(x)$ .

4. Find the derivative of the function  $y = \sqrt{x^2 - 1} \cdot \sec x$ .