Steps for Solving Related Rates Problems

1. Read and organize the information.
2. Define the *given* rate of change. This will be a $\frac{dy}{dt}$; that is, a change in y that depends on a change in t. (**look at the units** following the numbers to determine which quantities represent rates and which do not. Also pick a letter to replace y that represents the quantity in a memorable way. E.g $gallons/\text{min}$ could be $\frac{dV}{dt}$)
3. Define the *unknown* rate of change. This will be a $\frac{dx}{dt}$; that is , a change in x that depends on a change in t. (Again, replace x with a variable that best represents the quantity. E.g. $feet/\sec$ could be $\frac{dh}{dt}$ if it is a change in height with respect to time.
4. Write an equation (perhaps by using a formula below) that relates y to x. If you use a formula that relates more than just two variables, then use another relationship to replace all other variables, so that only x and y are related. (Note: we will differentiate this equation implicitly with respect to time t – **Do not substitute the given quantities in the equation yet**)
5. Differentiate both sides of the equation with respect to t and solve for the unknown rate of change after substituting in the given quantities.

Area Formulas (measured in square units)

Area of Rectangle: $A = lw$

Area of a Triangle: $A = \frac{1}{2}bh$

Area of a Circle: $A = \pi r^2$

Surface Area Formulas (measured in square units)

Surface Area of Rectanglar solid: $A = 2(lw + hw + hl)$

Surface Area of a Cylinder: $A = 2\pi rh + 2\pi r^2 h$

Surface Area of a Sphere: $A = 4\pi r^2$

Surface Area of a Cone: $A = \pi r\sqrt{r^2 + h^2}$

Volume Formulas (measured in cubic units)

Volume of a Rectangular Solid: $V = lwh$

Volume of a Cylinder: $V = \pi r^2 h$

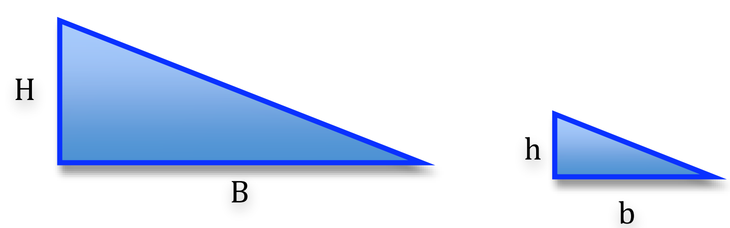
Volume of a Sphere: $V = \frac{4}{3}\pi r^3$

Volume of a Cone: $V = \frac{1}{3}\pi r^2 h$

Similar Triangles

Look for similar triangle in various shapes and use to establish a proportion to relate sides.

$$\frac{H}{B} = \frac{h}{b}$$



Pythagorean Theorem

For a right triangle with legs a and b and a hypotenus c: $a^2 + b^2 = c^2$

Right Triangle Trigonometry

Look for similar triangle in various shapes and use to establish a proportion to relate sides

$\sin x = \frac{opp}{hyp}$ $\cos x = \frac{adj}{hyp}$ $\tan x = \frac{opp}{adj}$

(note: $\csc x, \sec x,$ *and* $\cot x$ are just the reciprocals of these fractions, respectively)

1. If $x^2 + y^2 = 25$ and $\frac{dy}{dt} = 6$, find $\frac{dx}{dt}$ when $y = 4$.

2. Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of 1 **m/s**, *how fast* is the area of the spill increasing when the radius is 30 **m**?

3. A kite **100 ft above the ground** moves horizontally at a speed of 8 **ft/s**. *At what rate* is the angle between the string and the horizontal decreasing when 200 **ft** of string has been let out?

4. At noon, ship A is 150 **km** west of ship B. Ship A is sailing east at 35 **km/h** and ship B is sailing north at 25 **km/h**. *How fast* is the distance between the ships changing at 4:00pm?

5. A trough is 10 **ft** long and it's ends have the shape of a isosceles trangles that are 3 **ft** across at the top and have a height of 1 **ft**. If the trough is being filled with watter at a rate of 12 **ft³/min**, *how fast* is the water level rising when the water is 6 **inches** deep?

6. A water tank has a shape of an inverted circular cone with a base having radius 2 **m** and height 4 **m**. If the water is being pumped into the tank at a rate of 2 **m³/min**, find the *rate* at which the water level is rising when the water is 3 **m** deep.

7. A streetlight is mounted at the top of a 15 **ft** tall pole. A man 6 **ft** tall walks away from the pole with a speed of 5 **ft/s** along a straight path. *How fast* is the tip of his shadow moving when he is 40 **ft** from the pole?