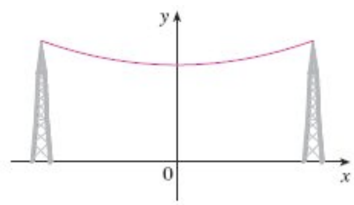
Definition of Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \coth x = \frac{\cosh x}{\sinh x}$$

The most common application of hyperbolic functions is that the curve of a hanging wire suspended between two points at the same height can be described by a catenary: $y = c + a\cosh\left(\frac{x}{a}\right)$



Illustrations of Hyperbolic Functions

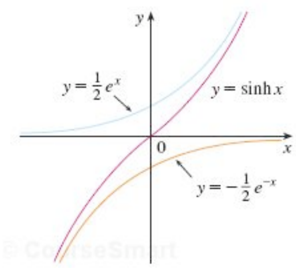


FIGURE 1
$y = \sinh x = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

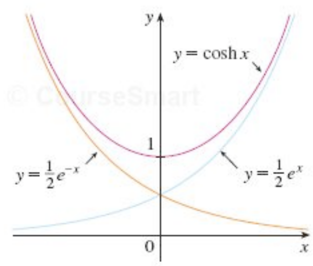


FIGURE 2
$y = \cosh x = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$

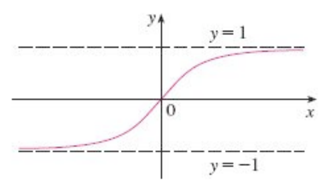
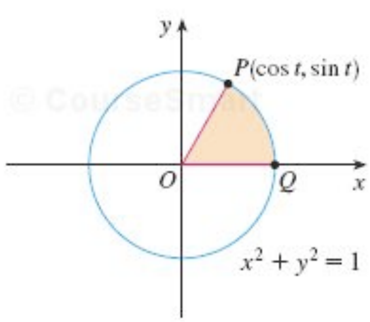


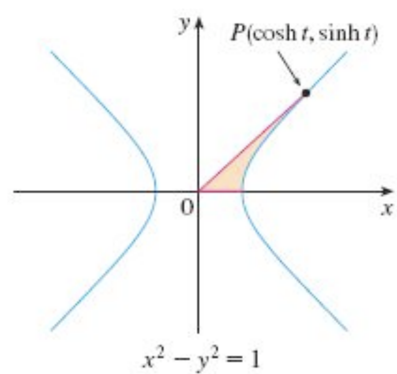
FIGURE 3
$y = \tanh x$

Why are these functions related to trigonometry?

The point $P(\cos t, \sin t)$ lies on the unit circle $x^2 + y^2 = 1$ because $\sin^2 t + \cos^2 t = 1$.

Similarly, the point $P(\cosh t, \sinh t)$ lies on the right branch of the hyperbola $x^2 - y^2 = 1$ because $\sinh^2 x - \cosh^2 = 1$.





In both cases, t represents twice the area of the shaded sector.

Hyperbolic Identities

$$\sinh(-x) = -\sinh x \qquad \cosh(-x) = \cosh x$$

$$\cosh^2 x - \sinh^2 x = 1 \qquad 1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$$

$$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$

1. Prove that $\cosh^2 x - \sinh^2 x = 1$.

2. Prove that $\cosh(2x) = \cosh^2 x + \sinh^2 x$

3. Find $\frac{d}{dx}(\sinh x)$.

4. Find $\frac{d}{dx}(\cosh x)$.

Derivatives of Hyperbolic Functions

$$\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

$$\frac{d}{dx}(\cosh x) = \sinh x \qquad \frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x \qquad \frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$$

Find the derivative. Simplify where possible.

5. $f(x) = x\cosh x + \sinh x$

6. $y = e^{\sinh x}$

7. $y = \sinh^2(\ln x)$