Note: A max or min value (whether it be absolute or local) is a y-value that occurs at some x-value.

Absolute Max and Min Definitions

A function f has an **absolute maximum** (or **global maximum**) at c if $f(c) \geq f(x)$ for all x in D, where D is the domain of f. The number $f(c)$ is called the **maximum value** of f on D.

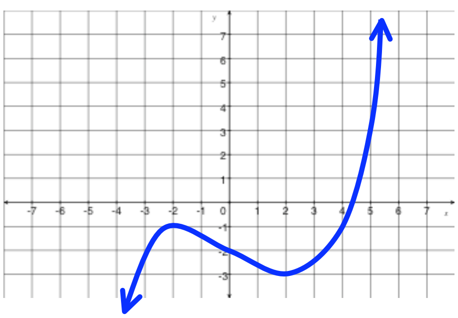
Similarly, a function f has an **absolute minimum** (or **global minimum**) at c if $f(c) \leq f(x)$ for all x in D, where D is the domain of f. In this case, the number $f(c)$ is called the **minimum value** of f on D.

The absolute maximum and absolute minimum of f are called the **extreme values** of f.

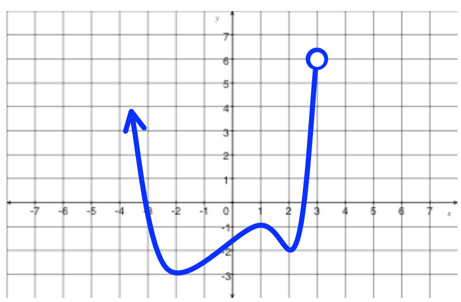
Local Max and Min Definitions

A function f has a **local maximum** (or **relative maximum**) at c if $f(c) \geq f(x)$ when x is near c. This means that $f(c) \geq f(x)$ for all x in some open interval containing c.

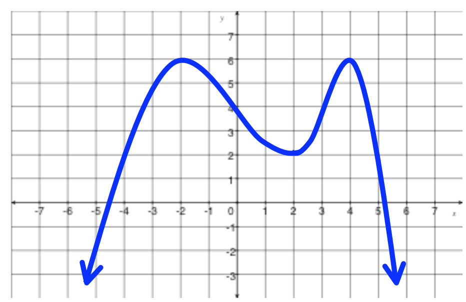
Similarly, a function f has a **local minimum** (or **relative minimum**) at c if $f(c) \leq f(x)$ when x is near c. This means that $f(c) \leq f(x)$ for all x in some open interval containing c.



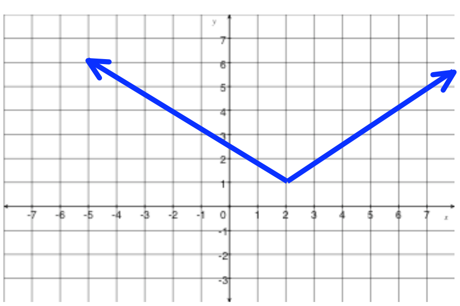
Local maximum(s):
Absolute maximum(s):
Local minimum(s):
Absolute minimums(s):



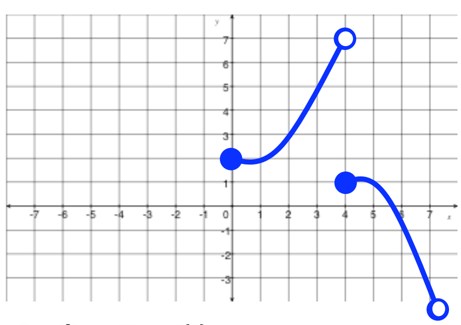
Local maximum(s):
Absolute maximum(s):
Local minimum(s):
Absolute minimums(s):



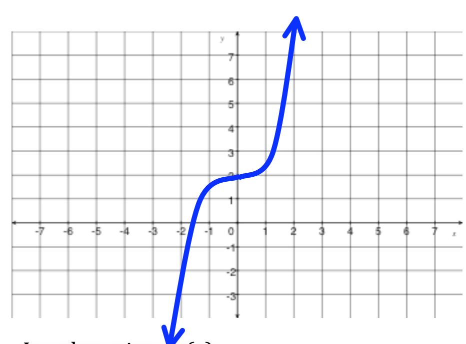
Local maximum(s):
Absolute maximum(s):
Local minimum(s):
Absolute minimums(s):



Local maximum(s):
Absolute maximum(s):
Local minimum(s):
Absolute minimums(s):



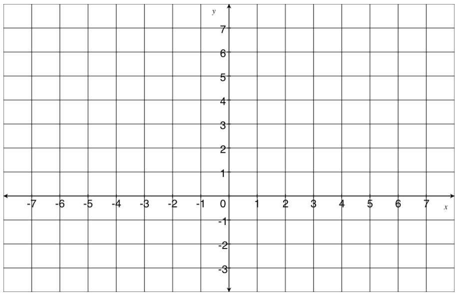
Local maximum(s):
Absolute maximum(s):
Local minimum(s):
Absolute minimums(s):



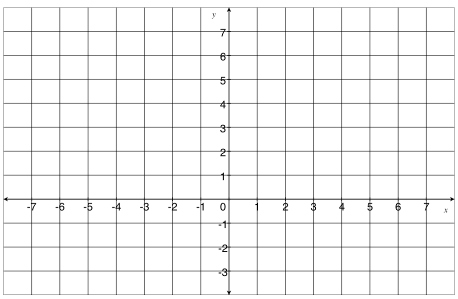
Local maximum(s):
Absolute maximum(s):
Local minimum(s):
Absolute minimums(s):

Sketch the graph of a function f that is continuous on $[-2, 3]$ and has the given properties:

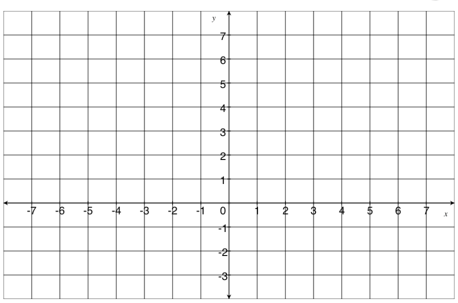
1. Absolute minimum at 0, absolute maximum at 3, relative maximum at 1.



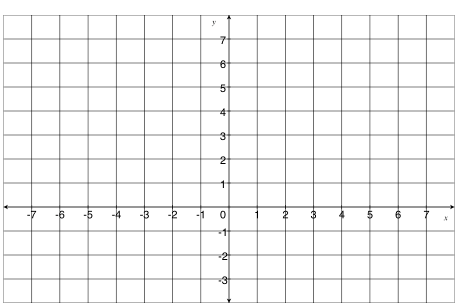
2. Sketch the graph of a function on $[0, 5]$ that has an absolute maximum, but no local minimum.



3. Sketch the graph of a function on $[0, 5]$ that has an absolute maximum at 2, but is not continuous at 2.



4. Sketch the graph of $f(x) = x^2 - 1$ for $|x| \le 2$ and use your sketch to find the absolute and local maximum and minimum values of f.



Extreme Value Theorem (E.V.T.)

If *f* is **continuous** on a **closed** interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in the interval $[a, b]$.

Note: This theorem is used to prove the existance of absolute maximum and absolute minimum values.

Fermat's Theorem

If *f* has a **local** maximum or minimum at c, and if $f'(c)$ **exists**, then $f'(c) = 0$

Note:
(1) This theorem helps us to establish a way to find very important numbers (called *critical numbers*).
(2) Also, note that the converse of this theorem is not always true.
That is, if $f'(c) = 0$, it is not always true that f has a local maximum or minimum at c.

Critical Numbers

A critical number of a function *f* is a number c in the domain of f such that $f'(c) = 0$ **OR** $f'(c)$ does not exist.
If f has a local maximum or minimum at c, then c is a critical number of f but if c is a critical number, it does **not** mean that $f'(c)$ is a local maximum or local minimum.

Find the critical numbers of the functions given below. (These values are where possible local maximums or minimums occur.)

5. $f(x) = x^3 + 2x + x$

6. $h(t) = t^{3/4} - 6t^{1/4}$

7. $y = \frac{1}{2}x + \cos^2 x$

The Closed Interval Method

To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

0. Note: The condition above that f is continuous on a closed interval guarantees that there are absolute maximum/minimum values on the closed interval by E.V.T. so first check that this condition is satisfied. Are conditions of E.V.T. satisfied? Yes? If yes, go onto step 1. If no, don't use this method to find absolute extrema. (Why not?)
1. Find the critical numbers of f in the open interval (a, b).
2. Find the y-values associated with the above critical numbers.
3. Find the y-values at the endpoints of the closed interval, namely a and b.
4. Compare all y-values. The largest y-value is the absolute maximum. The smallest y-value is the absolute minimum.

Find the **absolute** maximum and **absolute** minimum values of f on the given interval.

8. $f(x) = x^3 - 6x^2 + 9x + 2$, $[-1, 4]$

9. $f(x) = \dfrac{x^2 - 4}{x^2 + 1}$, $[-3, 2]$

10. $f(x) = x^2 e^{-3x}$, $[-2, 2]$