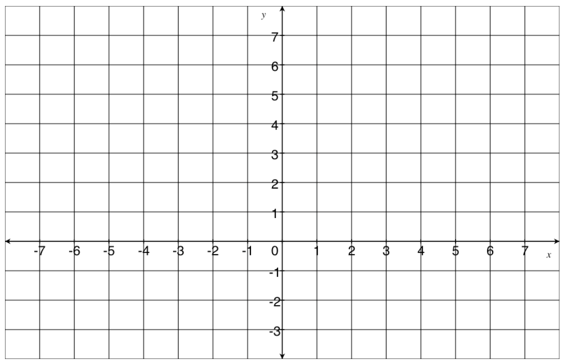
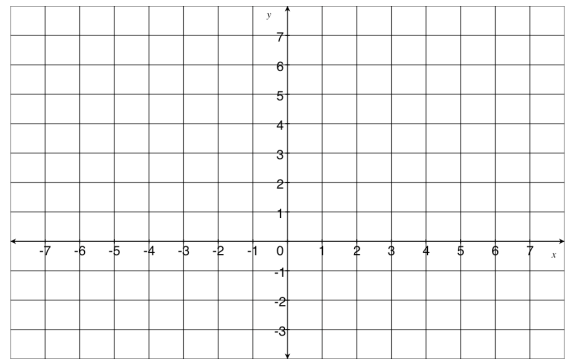
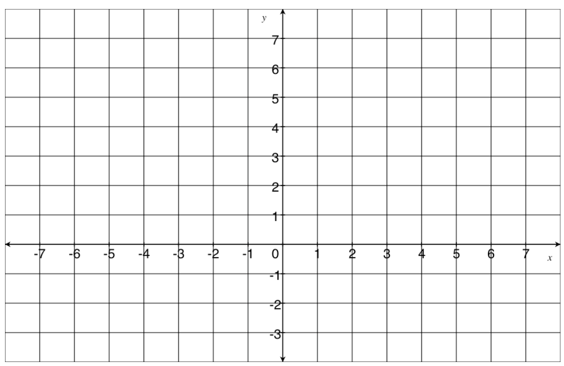
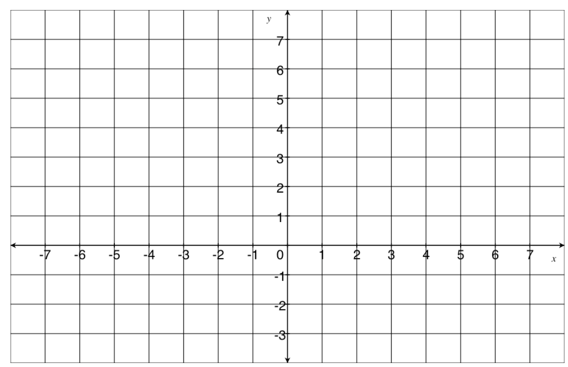
Draw four different functions f that are continuous on the closed interval $[a, b]$, differentiable on the open interval (a, b), and such that $f(a) = f(b)$.









Rolles Theorem

Let f be a function that satisfies the following three conditions:

1. f is continuous on the closed interval $[a, b]$
2. f is differentiable on the open interval (a, b)
3. $f(a) = f(b)$

Then there is a number c in (a, b) such that $f'(c) = 0$.

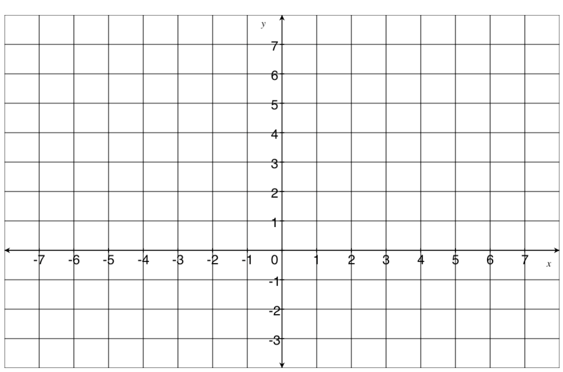
This means that if f is a continuous differentiable function that contains two points with equivalent y-values, then there exists some $x\text{-value}$ for which the derivative is 0.

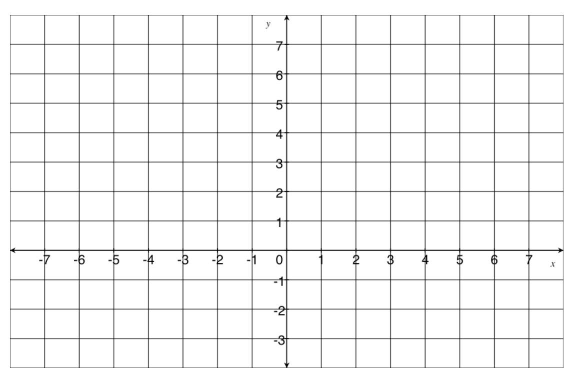
Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all the numbers c that satisfy the conclusion of Rolle's Theorem.

1. $f(x) = x^3 - x^2 - 6x + 2$ $\quad [0, 3]$

2. Let $f(x)=\tan x$. Show that $f(0)=f(\pi)$, but there is no number c in $(0,\pi)$ such that $f'(c)=0$. Why does this not contradict Rolle's Theorem?

Draw two different functions f that are continuous on the closed interval $[a,b]$, and differentiable on the open interval (a,b). Then label the average rate of change $\dfrac{f(b)-f(a)}{b-a}$ over the interval.





Notice how there is a number c in the interval (a,b), for which this average rate of change is equal to the instantaneous rate of change at c.

Mean Value Theorem

Let f be a function that satisfies the following conditions:

1. f is continuous on the closed intertval $[a, b]$
2. f is differentiable on the open interval (a, b)

Then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or equivalently

$$f(b) - f(a) = f'(c)(b - a)$$

This means that for a continuous and differentiable function, at some x-value, c, in the open interval (a, b), the instantaneous rate of change at that x-value, c, is the same as the average rate of change over the interval.

Theorem

If $f'(x) = 0$ for all x in an interval (a, b), then f is constant on (a, b).

Corollary

If $f'(x) = g'(x)$ for all x in an interval (a, b), then $f - g$ is constant on (a, b); that is, $f(x) = g(x) + c$, where c is a constant.

Verify that the funciton satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

3. $f(x) = \dfrac{x}{x+2}$ $\quad [1, 4]$

4. $f(x)=e^{2x}$ $\quad\quad [-3,0]$