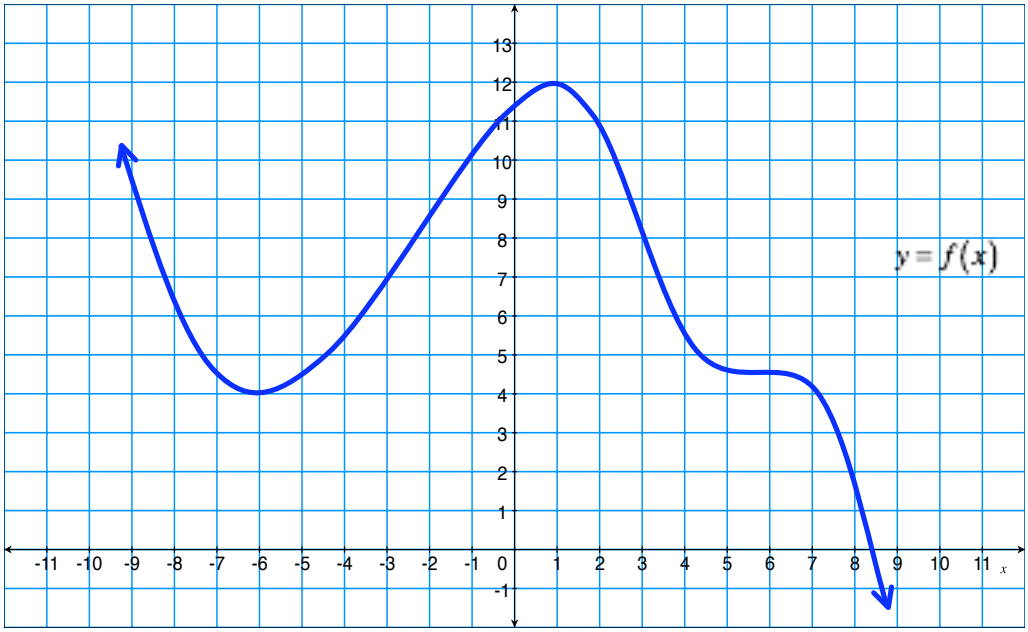
Use the following graph of the function f to answer questions 1 through 3



1. For which intervals is f increasing? What can be said about $f'(x)$ over these intervals?

2. For which intervals is f decreasing? What can be said about $f'(x)$ over these intervals?

Increasing/Decreasing Test

a) If $f'(x) > 0$ on an interval, then f is increasing on that interval.

b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

3. Find the critical points of f graphed above. Does a local maximum or minimum occur at any of these critical points? Describe the concavity of f around each critical point and determine the sign of $f'(x)$ on either side of each critical point.

The First Derivative Test

Suppose that c is a critical number of a continuous function f.

a) If f' changes from _____ to _____ at c, then f has a local **maximum** at c.

b) If f' changes from _____ to _____ at c, then f has a local **minimum** at c.

c) If f' doesn't change sign at c, then f has no local maximum or minimum at c.

Definition of Concavity

If the graph of f lies **above** all of its tangents on an interval I, then it is called **concave upward** on I.

If the graph of f lies **below** all of its tangents on an interval I, then it is called **concave downward** on I.

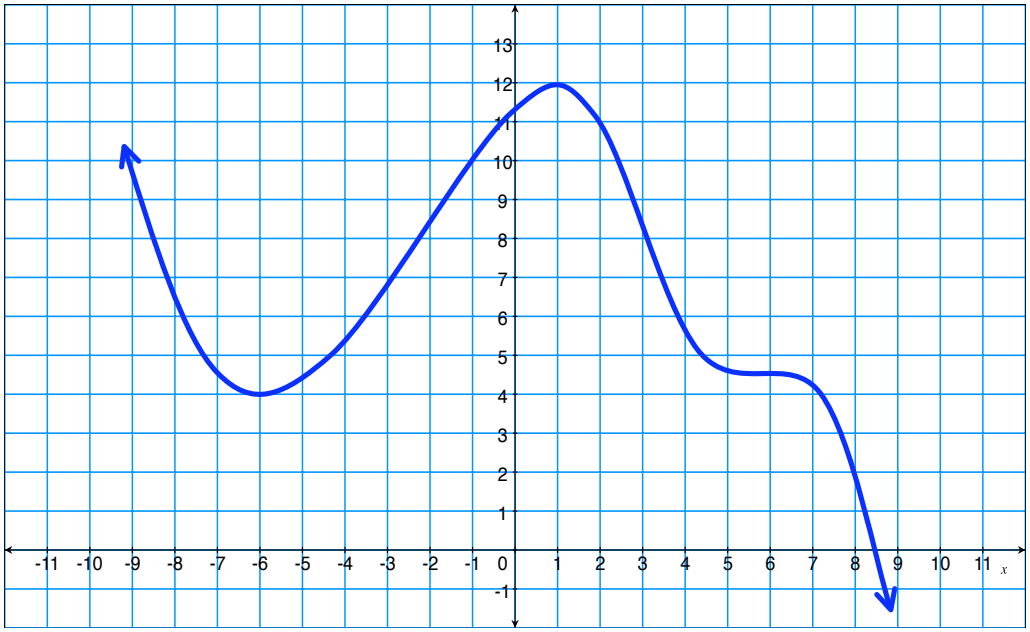
Concavity and Maximums and Minimums

Suppose that c is a critical number of a continuous function f.

a) If f is **concave upward** on an **open** interval that contains c, then f has a local **minimum** at c.

b) If f is **concave downward** on an **open** interval that contains c, then f has a local **maximum** at c.

Use the graph of f below to answer questions 4 and 5.



4. For which intevals is f concave upward? What can be said about $f'(x)$ over these intervals? Is $f'(x)$ increasing or decreasing on these intervals? What does this say about $f''(x)$?

5. For which intervals is f concave downward? What can be said about $f'(x)$ over these intervals? Is $f'(x)$ increasing or decreasing on these intervals? What does this say about $f''(x)$?

Definition of an Inflection Point

A point P on a curve $y = f(x)$ is called an inflection point if f is continuous there and the curve changes from concave upward to concave downward or ffrom concave downward to concave upward at P.

To find possible inflection points, set $f''(x) = 0$

Concavity Test

a) If $f''(x) > 0$ for all x in an interval I, then the graph of f is concave upward on I.

b) If $f''(x) < 0$ for all x in an interval I, then the graph of f is concave downward on I.

The Second Derivative Test

Suppose f' is **continuous** near c.

a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local _____ at c.

b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local _____ at c.

c is a critical point

For problems 6 and 7:

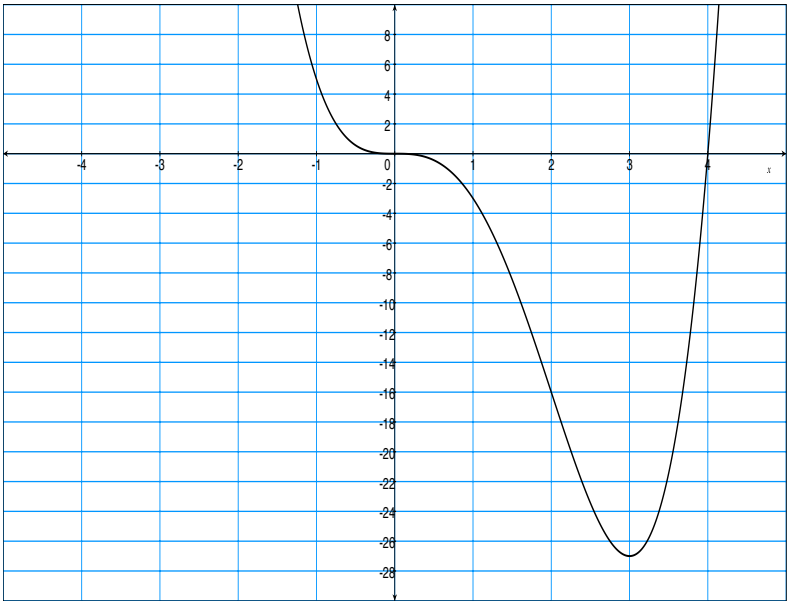
a) Find the intervals on which f is increasing or decreasing.

b) Find the local maximum and minimum values of f.

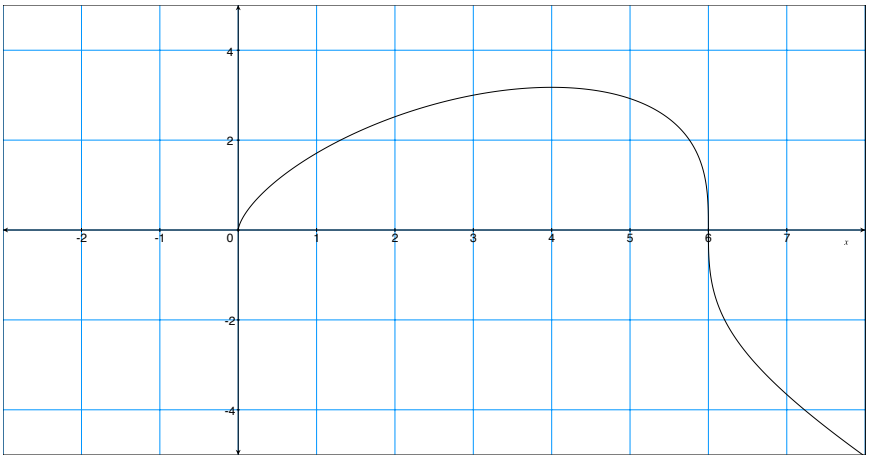
c) Find the intervals of concavity and the inflection points.

d) Use the information above to sketch a graph of f.

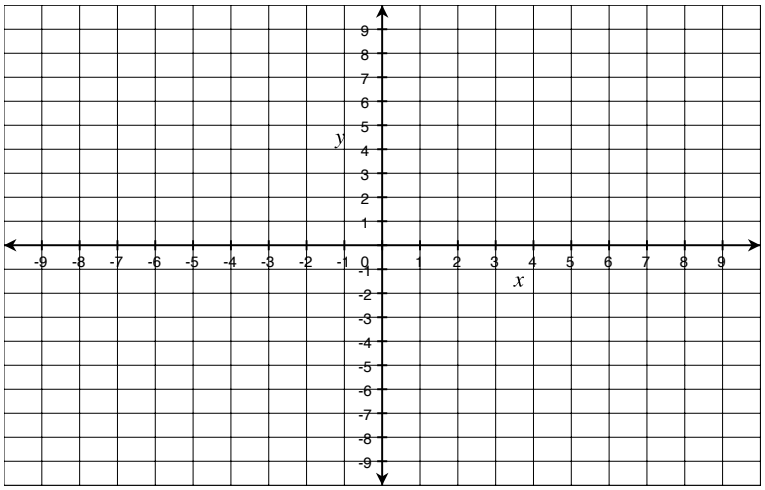
6. $f(x) = x^4 - 4x^3$



7. $f(x) = x^{2/3}(6-x)^{1/3}$

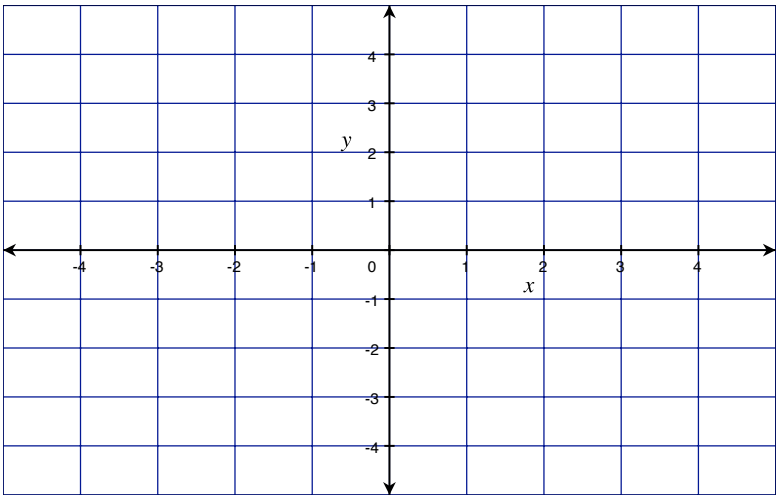


8. Use the first and second derivatives of $f(x) = e^{1/x}$, together with asymptotes, to sketch its graph.

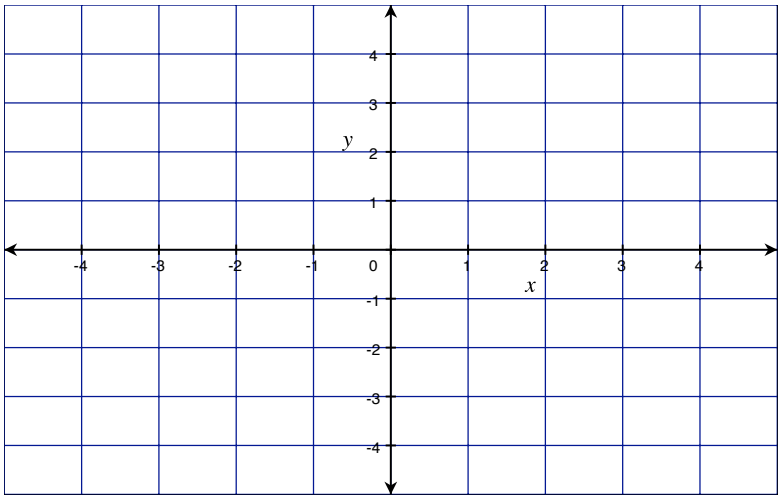


For problems 9 and 10, sketch a possible graph of a function f that satisfies all of the stated conditions.

9. $f'(x) > 0$ and $f''(x) > 0$ for all x



10. $f'(1)=f'(-1)=0$ and $f'(x)>0$ if $x<-1$



11. The graph of the first dervative f' of a function f is shown.

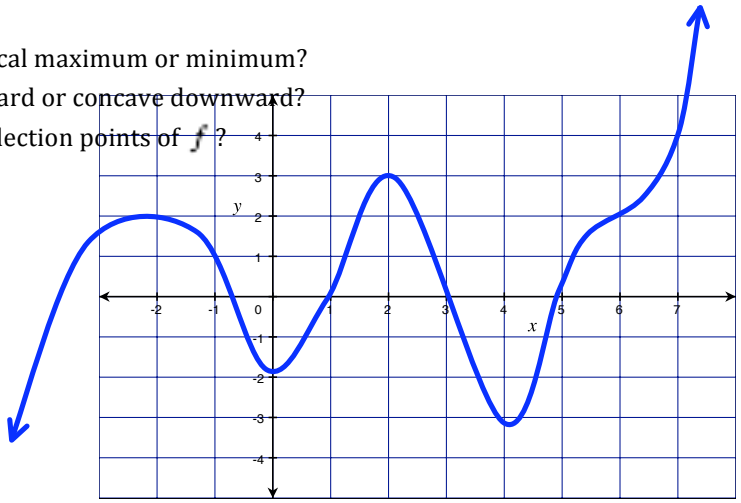
a. On what intervals is f increasing?

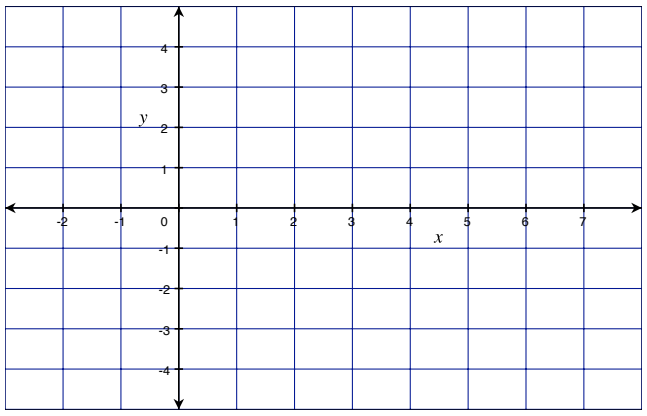
b. At what values of x does f have a local maximum or minimum?

c. On what intervals is f concave upward or concave downward?

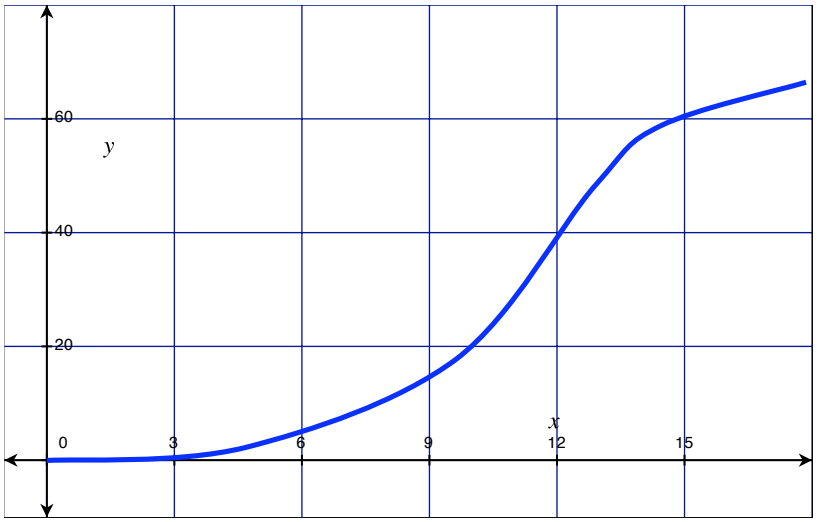
d. What are the x-coordinates of the inflection points of f?

e. Sketch a possible graph of f.





12. Below is a population graph for Cyprian honeybees raised in an apiary.



a. How does the rate of population increase change over time?

b. When is the rate of population increase the highest?

c. Over what intervals is P concave upward or concave downward?