Guidelines for Sketching a Curve

A. Find the **domain** of the function. Remember:
   a. Do not divide by zero.
   b. Do not take an even root of a negative number.
   c. Do not take a logarithm of a negative or zero.

B. Find the **intercepts** of the function.
   a. To find the y-intercept: plug 0 in for x.
   b. To find the x-intercept: set y equal to 0 and solve for x – if this is possible to do algebraically.

C. Determine if the function is **symmetric** or **periodic**.
   a. If $f(-x)=f(x)$, then f is an **even** function and is symmetric about the y-axis.
   b. If $f(-x)=-f(x)$, then f is an **odd** function and is symmetric about the origin.
   c. If there exists some constant p for which $f(x)=f(x+p)$ for all x-values, then f is aperiodic function, and the smallest such p is called the period.

D. Find any **asymptotes** or **holes**.
   a. To find vertical asymptotes (or holes), check the limits of the function as x approaches any values that are not in the function's domain.
   b. To find horizontal asymptotes, check the limits of the function as x approaches $\pm\infty$.
   c. To find *oblique* or *slant* asymptotes of a rational whose numerator's degree is larger than the denominator's degree, use long division.

E. Determine over which intervals the function is **increasing** and **decreasing**.
   a. Find $f'(x)$.
   b. Use a sign chart to test the sign of $f'(x)$.
   c. If $f'(x)>0$, then f is _____.
   d. If $f'(x)<0$, then f is _____.

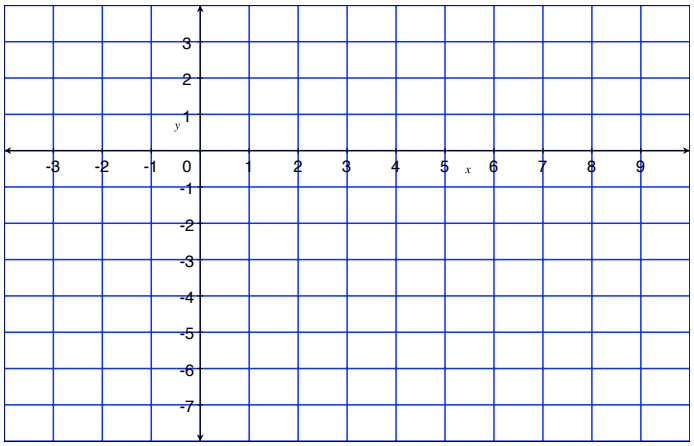
F. Find local **maximum** and **minimum** values.
   a. Find the critical number(s) of f. In other words find c where $f'(c)=0$ or $f'(c)$ doesn't exist.
   b. Use a sign chart for $f'(x)$ to determine whether a local maximum or local minimum or neither occurs at c.
      i. If $f'(x)$ changes from positive to negative at c (left to right), then there is a local maximum at c.
      ii. If $f'(x)$ changes from negative to positive at c (left to right), then there is a local minimum at c.
      iii. If $f'(x)$ doesn't change at c, then there isn't a local maximum or minimum at c.

G. Determine the **concavity** and **inflection** points of f.
   a. Find $f''(x)$.
   b. Use a sign chart to test the sign of $f''(x)$.
   c. If $f''(x)>0$ then f is concave _____.
   d. $f''(x)<0$ then f is concave _____.
   e. Inflection points occur where the direction of the concavity changes.

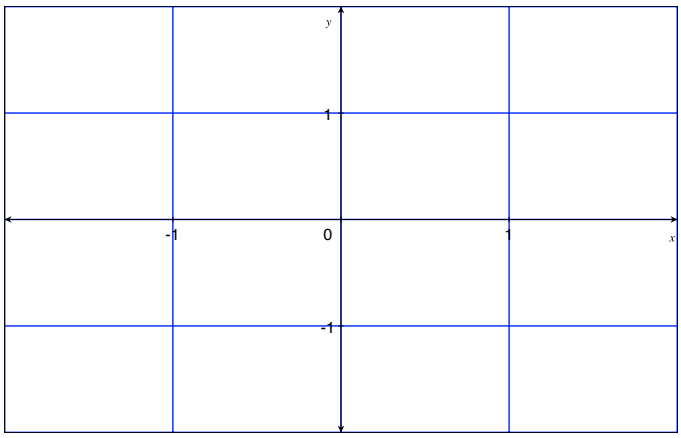
H. Sketch the curve .
   a. If you haven't found any of the outputs (y-values) for any of the significant x-values that you found above, do so now.
   b. Sketch the asymptotes as dashed lines.
   c. Label any intercepts, local maximums, local minimums and inflections points.
   d. Make sure the graph has the correct concavity.
   e. If additional accuracy is needed, determine the function's value at that point and maybe it's derivative there.

Use the guidelines to sketch the curve. Label any asymptotes, holes, local maximums, local minimums, intercepts and inflection points.

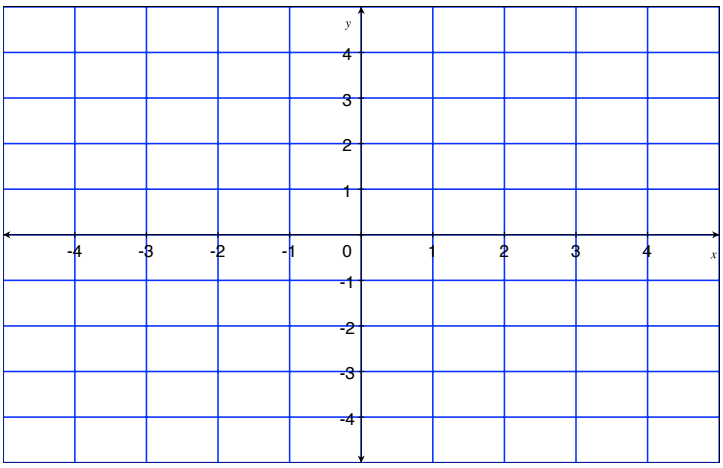
1. $x^{5/3} - 5x^{2/3}$



2. $y = x\sqrt{2 - x^2}$



3. $\frac{e^x}{x}$



4. $y = \dfrac{\ln x}{x^2}$