

Steps for Solving Optimization Problems

In the previous sections of Chapter 3 and 4, a function f is given and many of the problems ask about this function's rate of change (derivative). In this section, we look at problems where we are given a function that is a rate of change (derivative) and asked for the original function (antiderivative) that has that function as its derivative.

1. Let $f(x) = x^2$. Find a function, F , whose derivative is f , in other words, where $F' = f$. Is there more than one possible function F ?

Antiderivative

A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I .

Theorem

If F is an antiderivative of f on an interval I Then the most general antiderivative of f on I is $F(x) + C$ where C is an arbitrary constant.

2. $f(x) = \cos x$

3. $f(x) = \frac{1}{x}$

4. $f(x) = x^n$

5. $f(x) = \frac{1}{\sqrt{1-x^2}}$

Antidifferentiation Formulas

Function	Antiderivative
$cf(x)$	$cF(x) + c$
$f(x) + g(x)$	$F(x) + G(x) + C$
$x^n \ (n \neq -1)$	$\frac{x^{n+1}}{n+1} + C$
$\frac{1}{x}$	$\ln x + C$
e^x	$e^x + C$
$\cos x$	$\sin x + C$
$\sin x$	$-\cos x + C$
$\sec^2 x$	$\tan x + C$
$\sec x \tan x$	$\sec x + C$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + C$
$\frac{1}{1+x^2}$	$\tan^{-1} x + C$

Find $f(x)$ when:

6. $f'(x) = 2x - \frac{3}{x^4}$

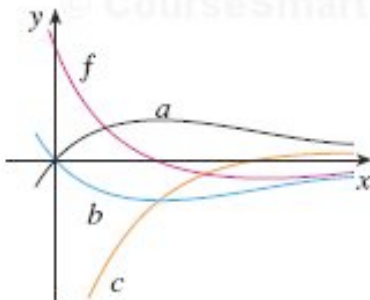
7. $f''(x) = 6x + \sin x$

8. $f''(x) = 20x^3 + 12x^2 + 4$

9. $f'(x) = \frac{x^2 - 1}{x}$, $f(1) = \frac{1}{2}$, $f(-1) = 0$

10. $f'(x) = e^x + 20(1+x^2)^{-1}$, $f(0) = -2$

11. The graph of a function f is shown. Which graph is an antiderivative of f and why?



12. Objects near the surface of the earth are subject to a gravitational force that produces a downward acceleration denoted by g . For motion close to the ground, we may assume that g is a constant: $g = 9.8 \text{ m/s}^2$ or 32 ft/s^2 . A ball is thrown upward with a speed of 48 ft/s from the edge of a cliff 432 ft above the ground.
- Find the ball's height above the ground t seconds later.

b. When does the ball reach its maximum height?

c. When does the ball hit the ground?